Revised Report on the Algorithmic Language ALGOL 60

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Dedicated to the Memory of WILLIAM TURANSKI

SUMMARY

The report gives a complete defining description of the international algorithmic language ALGOL 60. This is a language suitable for expressing a large class of numerical processes in a form sufficiently concise for direct automatic translation into the language of programmed automatic computers.

The introduction contains an account of the preparatory work leading up to the final conference, where the language was defined. In addition, the notions, reference language, publication language and hardware representations are explained.

In the first chapter, a survey of the basic constituents and features of the language is given, and the formal notation, by which the syntactic structure is defined, is explained.

The second chapter lists all the basic symbols, and the syntactic units known as identifiers, numbers and strings are defined. Further, some important notions such as quantity and value are defined.

The third chapter explains the rules for forming expressions and the meaning of these expressions. Three different types of expressions exist: arithmetic, Boolean (logical) and designational.

The fourth chapter describes the operational units of the language, known as statements. The basic statements are: assignment statements (evaluation of a formula), go to statements (explicit break of the sequence of execution of statements), dummy statements, and procedure statements (call for execution of a closed process, defined by a procedure declaration). The formation of more complex structures, having statement character, is explained. These include: conditional statements, for statements, compound statements, and blocks.

In the fifth chapter, the units known as declarations, serving for defining permanent properties of the units entering into a process described in the language, are defined.

The report ends with two detailed examples of the use of the language and an alphabetic index of definitions.

CONTENTS

INTRODUCTION

Background

After the publication of a preliminary report on the algorithmic language ALGOL,[1,2] as prepared at a conference in Zurich in 1958, much interest in the ALGOL language developed.

As a result of an informal meeting held at Mainz in November 1958, about forty interested persons from several European countries held an ALGOL implementation conference in Copenhagen in February 1959. A "hardware group" was formed for working cooperatively right down to the level of the paper tape code. This conference also led to the publication by Regnecentralen, Copenhagen, of an *ALGOL Bulletin*, edited by Peter Naur, which served as a forum for further discussion. During the June 1959 ICIP Conference in Paris several meetings, both formal and informal ones, were held. These meetings revealed some misunderstandings as to the intent of the group which was primarily responsible for the formulation of the language, but at the same time made it clear that there exists a wide appreciation of the effort involved. As a result of the discussions it was decided to hold an international meeting in January 1960 for improving the ALGOL language and preparing a final report. At a European ALGOL Conference in Paris in November 1959 which was attended by about fifty people, seven European representatives were selected to attend the January 1960 Conference, and they represent the following organizations: Association Française de Calcul, British Computer Society, Gesellschaft für Angewandte Mathematik und Mechanik, and Nederlands Rekenmachine Genootschap. The seven representatives held a final preparatory meeting at Mainz in December 1959.

Meanwhile, in the United States, anyone who wished to suggest changes or corrections to ALGOL was requested to send his comments to the *Communications of the ACM*, where they were published. These comments then became the basis of consideration for changes in the ALGOL language. Both the SHARE and USE organizations established ALGOL working groups, and both organizations were represented on the ACM Committee on Programming Languages. The ACM Committee met in Washington in November 1959 and considered all comments on ALGOL that had been sent to the ACM *Communications*. Also, seven representatives were selected to attend the January 1960 international conference. These seven representatives held a final preparatory meeting in Boston in December 1959.

January 1960 Conference

The thirteen representatives,[3] from Denmark, England, France, Germany, Holland, Switzerland, and the United States, conferred in Paris from January 11 to 16, 1960.

Prior to this meeting a completely new draft report was worked out from the preliminary report and the recommendations of the preparatory meetings by Peter Naur and the conference adopted this new form as the basis for its report. The Conference then proceeded to work for agreement on each item of the report. The present report represents the union of the Committee's concepts and the intersection of its agreements.

April 1962 Conference [Edited by M. Woodger]

A meeting of some of the authors of ALGOL 60 was held on April *2–3,* 1962 in Rome, Italy, through the facilities and courtesy of the International Computation Centre. The following were present:

Authors	*Advisers*	*Observer*
F. L. Bauer	M. Paul	W. L. van der Poel (Chairman, IFIP TC 2.1 Working Group ALGOL)
J. Green	R. Franciotti	
C. Katz	P. Z. Ingerman	
R. Kogon (representing J. W. Backus)		
P. Naur		
K. Samelson	G. Seegmüller	
J. H. Wegstein	R. E. Utman	
A. van Wijngaarden		
M. Woodger	P. Landin	

The purpose of the meeting was to correct known errors in, attempt to eliminate apparent ambiguities in, and otherwise clarify the ALGOL 60 Report. Extensions to the language were not considered at the meeting. Various proposals for correction and clarification that were submitted by interested parties in response to the Questionnaire in *ALGOL Bulletin* No. 14 were used as a guide.

This report* constitutes a supplement to the ALGOL 60 Report which should resolve a number of difficulties therein. Not all of the questions raised concerning the original report could be resolved. Rather than risk hastily drawn conclusions on a number of subtle points, which might create new ambiguities, the committee decided to report only those points which they unanimously felt could be stated in clear and unambiguous fashion.

Questions concerned with the following areas are left for further consideration by Working Group 2.1 of IFIP, in the expectation that current work on advanced pro-

* [EDITOR'S NOTE. The present edition follows the text which was approved by the Council of IFIP. Although it is not clear from the Introduction, the present version is the original report of the January 1960 conference modified according to the agreements reached during the April 1962 conference. Thus the report mentioned here is incorporated in the present version. The modifications touch the original report in the following sections: Changes of text: 1 with footnote; 2.1 footnote; 2.3; 2.7; 3.3.3; 3.3.4.2; 4.1.3; 4.2.3; 4.2.4; 4.3.4; 4.7.3; 4.7.3.1; 4.7.3.3; 4.7.5.1; 4.7.5.4; 4.7.6; 5; 5.3.3; 5.3.5; 5.4.3; 5.4.4; 5.4.5. Changes of syntax: 3.4.1; 4.1.1; 4.2.1; 4.5.1.]

[1] Preliminary report—International Algebraic Language. *Comm. ACM 1*, 12 (1958), 8.

[2] Report on the Algorithmic Language ALGOL by the ACM Committee on Programming Languages and the GAMM Committee on Programming, edited by A. J. Perlis and K. Samelson. *Num. Math. 1* (1959), 41–60.

[3] William Turanski of the American group was killed by an automobile just prior to the January 1960 Conference.

gramming languages will lead to better resolution:

1. Side effects of functions
2. The call by name concept
3. **own**: static or dynamic
4. For statement: static or dynamic
5. Conflict between specification and declaration

The authors of the ALGOL 60 Report present at the Rome Conference, being aware of the formation of a Working Group on ALGOL by IFIP, accepted that any collective responsibility which they might have with respect to the development, specification and refinement of the ALGOL language will from now on be transferred to that body.

This report has been reviewed by IFIP TC 2 on Programming Languages in August 1962 and has been approved by the Council of the International Federation for Information Processing.

As with the preliminary ALGOL report, three different levels of language are recognized, namely a Reference Language, a Publication Language and several Hardware Representations.

REFERENCE LANGUAGE

1. It is the working language of the committee.
2. It is the defining language.
3. The characters are determined by ease of mutual understanding and not by any computer limitations, coders notation, or pure mathematical notation.
4. It is the basic reference and guide for compiler builders.
5. It is the guide for all hardware representations.
6. It is the guide for transliterating from publication language to any locally appropriate hardware representations.
7. The main publications of the ALGOL language itself will use the reference representation.

PUBLICATION LANGUAGE

1. The publication language admits variations of the reference language according to usage of printing and handwriting (e.g., subscripts, spaces, exponents, Greek letters).
2. It is used for stating and communicating processes.
3. The characters to be used may be different in different countries, but univocal correspondence with reference representation must be secured.

HARDWARE REPRESENTATIONS

1. Each one of these is a condensation of the reference language enforced by the limited number of characters on standard input equipment.
2. Each one of these uses the character set of a particular computer and is the language accepted by a translator for that computer.
3. Each one of these must be accompanied by a special set of rules for transliterating from Publication or Reference language.

For transliteration between the reference language and a language suitable for publications, among others, the following rules are recommended.

Reference Language	*Publication Language*
Subscript bracket [ ]	Lowering of the line between the brackets and removal of the brackets
Exponentiation ↑	Raising of the exponent
Parentheses ( )	Any form of parentheses, brackets, braces
Basis of ten ₁₀	Raising of the ten and of the following integral number, inserting of the intended multiplication sign

DESCRIPTION OF THE REFERENCE LANGUAGE

> Was sich überhaupt sagen lässt, lässt sich klar sagen; und wovon man nicht reden kann, darüber muss man schweigen.
> LUDWIG WITTGENSTEIN.

Structure of the Language

As stated in the introduction, the algorithmic language as three different kinds of representations—reference, ardware, and publication—and the development deribed in the sequel is in terms of the reference reprentation. This means that all objects defined within the nguage are represented by a given set of symbols—and is only in the choice of symbols that the other two :Presentations may differ. Structure and content must e the same for all representations.

The purpose of the algorithmic language is to describe omputational processes. The basic concept used for the escription of calculating rules is the well-known arithmetic expression containing as constituents numbers, variables, and functions. From such expressions are compounded, by applying rules of arithmetic composition, self-contained units of the language—explicit formulae—called assignment statements.

To show the flow of computational processes, certain nonarithmetic statements and statement clauses are added which may describe, e.g., alternatives, or iterative repetitions of computing statements. Since it is necessary for the function of these statements that one statement refer to another, statements may be provided with labels. A sequence of statements may be enclosed between the statement brackets **begin** and **end** to form a compound statement.

Statements are supported by declarations which are not themselves computing instructions but inform the translator of the existence and certain properties of objects appearing in statements, such as the class of numbers taken on as values by a variable, the dimension of an array of numbers, or even the set of rules defining a function. A sequence of declarations followed by a sequence of statements and enclosed between **begin** and **end** constitutes a block. Every declaration appears in a block in this way and is valid only for that block.

A program is a block or compound statement which is not contained within another statement and which makes no use of other statements not contained within it.

In the sequel the syntax and semantics of the language will be given.[4]

1.1. Formalism for Syntactic Description

The syntax will be described with the aid of metalinguistic formulae.[5] Their interpretation is best explained by an example

⟨ab⟩ ::= (| [| ⟨ab⟩ (| ⟨ab⟩⟨d⟩

Sequences of characters enclosed in the brackets ( ) represent metalinguistic variables whose values are sequences of symbols. The marks ::= and | (the latter with the meaning of **or)** are metaliriguistic connectives. Any mark in a formula, which is not, a variable or a connective, denotes itself (or the class of marks which are similar to it). Juxtaposition of marks and/or variables in a formula signifies juxtaposition of the sequences denoted. Thus the formula above gives **a** recursive rule for the formation of values of' the variable **(ah).** It indicates that (ab) may have the value (or [or that given some legitimate value of ⟨ab⟩, another may br formed by following it with the character (or by following it. with some value of the variable ⟨d⟩. If the values of ⟨d⟩ are the decimal digits, some values **of** ⟨ab⟩ are:

[(((1(37(
(12345(
(((
[86

In order to facilitate the study, the symbols used for distinguishing the metalinguistic variables (i.e. the sequences **of** characters appearing within the brackets ( ) as ab in the above example) have been chosen to be words describing approximately the nature of the corresponding variable. Where words which have appeared in this manner arc used elsewhere in the text they will refer to the corresponding syntactic definition. In addition **sonic** formulae have been given in more than one place.

Definition:

⟨empty⟩ ::=
(i.e. the null string of symbols).

[4] Whenever the precision of arithmetic is stated as being in general not specified, or the outcome of a certain process is left undefined or said to be undefined, this is to be interpreted in the sense that a program only fully defines a computational process if the accompanying information specifies the precision assumed, the kind of arithmetic assumed, and the course of action to be taken in all such cases as may occur during the execution of the computation.

[5] Cf. J. W. Backus, The syntax and semantics of the proposed international algebraic language of the Zürich ACM-GAMM conference. Proc. Internat. Conf. Inf. Proc., UNESCO, Paris, June 1959.

2. Basic Symbols, Identifiers, Numbers, and Strings. Basic Concepts.

The reference language is built up from the following basic symbols:

(basic symbol) ::= (letter)/(digit)](logical value)/(delimiter)

2.1. Letters

(letter⟩ ::= $a|b|c|d|e|f|g|h|i|j|k|l|m|n|o|p|q|r|s|t|u|v|w|x|y|z|$
$A|B|C|D|E|F|G|H|I|J|K|L|M|N|O|P|Q|R|S|T|U|V|W|X|Y|Z$

This alphabet may arbitrarily be restricted, or extended with any other distinctive character (i.e. character not coinciding with any digit, logical value or delimiter).

Letters do not have individual meaning. They are used for forming identifiers and strings[6] (cf. sections 2.4. Identifiers, **2.6.** Strings).

2.2.1. Digits

(digit) ::= 0|1|2|3|4|5|6|7|8|9

Digits are used for forming numbers, identifiers, and strings.

2.2.2. Logical Values

(logical value⟩ ::= **true**|**false**

The logical values have a fixed obvious meaning.

2.3. Delimiters

⟨delimiter⟩ ::= ⟨operator⟩|⟨separator⟩|⟨bracket⟩|⟨declarator⟩
(specificator⟩
(operator) ::= ⟨arithmetic operator⟩|⟨relational operator⟩|
⟨logical operator)|⟨sequential operator)
(arithmetic operator') ::= $+|-|\times|/|\div|\uparrow$
(relational operator⟩ ::= $<|\leqq|=|\geqq|>|\neq$
(logical operator⟩ ::= $\equiv|\supset|\vee|\wedge|\neg$
⟨sequential operator⟩ ::= **go to**|**if**|**then**|**else**|**for**|**do**[7]
⟨separator⟩ ::= ,|.|$_{10}$|:|;|:=|␣|**step**|**until**|**while**|**comment**
⟨bracket⟩ ::= (|)|[|]|‘|’|**begin**|**end**
⟨declarator⟩ ::= **own**|**Boolean**|**integer**|**real**|**array**|**switch**|
procedure
⟨specificator⟩ ::= **string**|**label**|**value**

Delimiters have **a** fixed meaning which for the most part is obvious or else will be given at the appropriate place in the sequel.

Typographical features such as blank space or change to a new line have no significance in the reference language. They may, however, be used freely for facilitating reading.

For the purpose of including text among the symbols of

[6] It should be particularly noted that throughout the reference language underlining [in typewritten copy; boldface type in printed copy—Ed.] is used for defining independent basic symbols (see sections 2.2.2 and 2.3). These are understood to have no relation to the individual letters of which they are composed. Within the present report [not including headings—Ed.], boldface will be used for no other purpose.

[7] **do** is used in for statements. It has no relation whatsoever to the *do* of the preliminary report, which is not included in ALGOL 60.

a program the following "comment" conventions hold:

The sequence of basic symbols:	*is equivalent to*
; **comment** ⟨any sequence not containing ;⟩;	;
begin comment ⟨any sequence not containing ;⟩;	**begin**
end ⟨any sequence not containing **end** or ; or **else**⟩	**end**

By equivalence is here meant that any of the three structures shown in the left-hand column may be replaced, in any occurrence outside of strings, by the symbol shown on the same line in the right-hand column without any effect on the action of the program. It is further understood that the comment structure encountered first in the text when reading from left to right has precedence in being replaced over later structures contained in the sequence.

2.4. Identifiers

2.4.1. Syntax

⟨identifier⟩ ::= ⟨letter⟩|⟨identifier⟩⟨letter⟩|⟨identifier⟩⟨digit⟩

2.4.2. Examples

q
Soup
V17a
a34kTMNs
MARILYN

2.4.3. Semantics

Identifiers have no inherent meaning, but serve for the entification **of** simple variables, arrays, labels, switches, ıd procedures. They may be chosen freely (cf., however, ction 3.2.4. Standard Functions).

The same identifier cannot be used to denote two fferent quantities except when these quantities have sjoint scopes as defined by the declarations **of** the pro-'am (cf. section *2.7.* Quantities, Kinds and Scopes, and ction 5. Declarations).

2.5. Numbers

2.5.1. Syntax

ınsigned integer⟩ ::= ⟨digit⟩|⟨unsigned integer⟩⟨digit⟩
nteger⟩ ::= ⟨unsigned integer⟩|+⟨unsigned integer⟩|
  −⟨unsigned integer⟩
decimal fraction⟩ ::= .⟨unsigned integer⟩
exponent part⟩ ::= $_{10}$⟨integer⟩
decimal number⟩ ::= ⟨unsigned integer⟩|⟨decimal fraction⟩|
  ⟨unsigned integer⟩⟨decimal fraction⟩
unsigned number⟩ ::= ⟨decimal number⟩|⟨exponent part⟩|
  ⟨decimal number⟩⟨exponent part⟩
number⟩ ::= ⟨unsigned number⟩|+⟨unsigned number⟩|
  −⟨unsigned number⟩

2.5.2. Examples

0	−200.084	−.083$_{10}$−02
177	+07.43$_{10}$8	−$_{10}$7
.5384	9.34$_{10}$+10	$_{10}$−4
+0.7300	2−$_{10}$4	+$_{10}$+5

2.5.3. Semantics

Decimal numbers have their conventional meaning. The exponent part is a scale factor expressed as an integral ower of 10.

2.5.4. Types

Integers are of type **integer**. All other numbers are of type **real** (cf. section 5.1. Type Declarations).

2.6. Strings

2.6.1. Syntax

⟨proper string⟩ ::= ⟨any sequence of basic symbols not containing
  ‘ or ’⟩|⟨empty⟩
⟨open string⟩ ::= ⟨proper string⟩|‘⟨open string⟩’|
  ⟨open string⟩⟨open string⟩
⟨string⟩ ::= ‘⟨open string⟩’

2.6.2. Examples

‘5k,,−‘[[[‘∧=/:’Tt’’
‘.. This ␣ is ␣ a ␣ ‘string’’

2.6.3. Semantics

In order to enable the language to handle arbitrary sequences of basic symbols the string quotes ‘ and ’ are introduced. The symbol ␣ denotes a space. It has no significance outside strings.

Strings are used as actual parameters of procedures (cf. sections 3.2. Function Designators and 4.7. Procedure Statements).

2.7. Quantities, Kinds and Scopes

The following kinds of quantities are distinguished: simple variables, arrays, labels, switches, and procedures.

The scope of a quantity is the set of statements and expressions in which the declaration of the identifier associated with that quantity is valid. For labels see section 4.1.3.

2.8. Values and Types

A value is an ordered set of numbers (special case: a single number), an ordered set of logical values (special case: a single logical value), or a label.

Certain of the syntactic units are said to possess values. These values will in general change during the execution of the program. The values of expressions and their constituents are defined in section :<.The value of an array identifier is the ordered set of values of the corresponding array of subscripted variables (cf. section 3.1.4.1).

The various "types" (**integer, real, Boolean**) basically denote properties of values. The types associated with syntactic units refer to the values of these units.

3. Expressions

In the language the primary constituents **of** the programs describing algorithmic processes are arithmetic, Boolean, and designational expressions. Constituents of these expressions, except for certain delimiters, are logical values, numbers, variables, function designators, and elementary arithmetic, relational, logical, and sequential operators. Since the syntactic definition of both variables and function designators contains expressions, the definition of expressions, and their constituents, is necessarily recursive.

⟨expression⟩ ::= ⟨arithmetic expression⟩|⟨Boolean expression⟩|
  ⟨designational expression⟩

3.1. VARIABLES

3.1.1. Syntax

⟨variable identifier⟩ ::= ⟨identifier⟩
⟨simple variable⟩ ::= ⟨variable identifier⟩
⟨subscript expression⟩ ::= ⟨arithmetic expression⟩
⟨subscript list⟩ ::= ⟨subscript expression⟩|⟨subscript list⟩,
    ⟨subscript expression⟩
⟨array identifier⟩ ::= ⟨identifier⟩
⟨subscripted variable⟩ ::= ⟨array identifier⟩[⟨subscript list⟩]
⟨variable⟩ ::= ⟨simple variable⟩|⟨subscripted variable⟩

3.1.2. Examples

epsilon
detA
a17
Q[7,2]
x[*sin*(*n*×*pi*/2),*Q*[3,*n*,4]]

3.1.3. Semantics

A variable is a designation given to a single value. This value may be used in expressions for forming other values and may be changed at will by means of assignment statements (section **4.2**). The type of the value of a particular variable is defined in the declaration for the variable itself (cf. section 5.1. Type Declarations) or for the corresponding array identifier (cf. section *5.2.* Array Declarations).

3.1.4. Subscripts

3.1.4.1. Subscripted variables designate values which are components of multidimensional arrays (cf. section *5.2.* Array Declarations). Each arithmetic expression of the subscript list occupies one subscript position of the subscripted variable, and is called a subscript,. The complete list of subscripts is enclosed in the subscript, brackets [ ]. The array component referred to by a subscripted variable is specified by the actual numerical value of its subscripts (cf. section 3.3. Arithmetic Expressions).

3.1.4.2. Each subscript position acts like a variable of type **integer** and the evaluation of the subscript, is understood to be equivalent t.o an assignment, to this fictitious variable (cf. section 4.2.4). The value of the subscripted variable is defined only if the value of the subscript expression is within the subscript bounds of the array (cf. section 5.2. Array Declarations).

3.2. FUNCTION DESIGNATORS

3.2.1. Syntax

⟨procedure identifier⟩ ::= ⟨identifier⟩
⟨actual parameter⟩ ::= ⟨string⟩|⟨expression⟩|⟨array identifier⟩|
    ⟨switch identifier⟩|⟨procedure identifier⟩
⟨letter string⟩ ::= ⟨letter⟩|⟨letter siring⟩⟨leiter⟩
⟨parameter delimiter⟩ ::= ,|)⟨letter string⟩:(
⟨actual parameter list⟩ ::= ⟨actual parameter⟩|
    ⟨actual parameter list⟩⟨parameter delimiter⟩
    ⟨actual parameter⟩
⟨actual parameter part⟩ ::= ⟨empty⟩|(⟨actual parameter list⟩)
⟨function designator⟩ ::= ⟨procedure identifier⟩
    ⟨actual parameter part⟩

3.2.2. Examples

sin(*a*−*b*)
J(*v*+*s*,*n*)
R
S(*s*−5)Temperature:(*T*)Pressure:(P)
Compile(' := ')Stack:(&)

3.2.3. Semantics

Function designators define single numerical or logical values, which result through the application of given sets of rules defined by a procedure declaration (cf. section **5.4.** Procedure Declarations) to fixed sets of actual parameters. The rules governing specification of actual parameters are given in section 4.7. Procedure Statements. Not every procedure declaration defines the value of a function designator.

3.2.4. Standard functions

Certain identifiers should be reserved for the standard functions of analysis, which will be expressed as procedures. It is recommended that this reserved list should contain:

abs(E)	for the modulus (absolute value) of the value of the expression E
sign(E)	for the sign of the value of E(+1 for $E>0$, 0 for $E=0$, −1 for $E<0$)
sqrt(E)	for the square root of the value of E
sin(E)	for the sine of the value of E
cos(E)	for the cosine of the value of E
arctan(E)	for the principal value of the arctangent of the value of E
ln(E)	for the natural logarithm of the value of E
exp(E)	for the exponential function of the value of E (e^E).

These functions are all understood to operate indifferently on arguments both of type **real** and **integer**. They will all yield values of type **real**, except for *sign*(E) which will have values of type **integer**. In a particular representation these functions may be available without, explicit declarations (cf. section 5. Declarations).

3.2.5. Transfer functions

It is understood that transfer functions between any pair of quantities and expressions map be defined. Among the standard functions it is recommended that, there be one, namely,

entier(E),

which "transfers" an expression of real type to one of integer type, and assigns to it the value which is the largest integer not greater than the value of E.

3.3. ARITHMETIC EXPRESSIONS

3.3.1. Syntax

⟨adding operator⟩ ::= +|−
⟨multiplying operator⟩ ::= ×|/|÷
⟨primary⟩ ::= ⟨unsigned number⟩|⟨variable⟩|
    ⟨function designator⟩|(⟨arithmetic expression⟩)
⟨factor⟩ ::= ⟨primary⟩|⟨factor⟩↑⟨primary⟩
⟨term⟩ ::= ⟨factor⟩|⟨term⟩⟨multiplying operator⟩⟨factor⟩
⟨simple arithmetic expression⟩ ::= ⟨term⟩|
    ⟨adding operator⟩⟨term⟩|⟨simple arithmetic expression⟩
    ⟨adding operator⟩⟨term⟩
⟨if clause⟩ ::= **if** ⟨Boolean expression⟩**then**
⟨arithmetic expression⟩ ::= ⟨simple arithmetic expression⟩|
    ⟨if clause⟩⟨simple arithmetic expression⟩**else**
    ⟨arithmetic expression⟩

3.3.2. Examples

Primaries:

$7.394_{10}-8$
sum
$w[i+2,8]$
$cos(y+z\times 3)$
$(a-3/y+vu\uparrow 8)$

Factors:

omega
$sum\uparrow cos(y+z\times 3)$
$7.394_{10}-8\uparrow w[i+2,8]\uparrow(a-3/y+vu\uparrow 8)$

Terms:

U
$omega\times sum\uparrow cos(y+z\times 3)/7.394_{10}-8\uparrow w[i+2,8]\uparrow(a-3/y+vu\uparrow 8)$

Simple arithmetic expression:

$U-Yu+omega\times sum\uparrow cos(y+z\times 3)/7.394_{10}-8\uparrow w[i+2,8]\uparrow(a-3/y+vu\uparrow 8)$

Arithmetic expressions:

$w\times u-Q(S+Cu)\uparrow 2$
if $q>0$ **then** $S+3\times Q/A$ **else** $2\times S+3\times q$
if $a<0$ **then** $U+V$ **else if** $a\times b>17$ **then** U/V **else if** $k\neq y$ **then** V/U **else** 0
$a\times sin(omega\times t)$
$0.57_{10}12\times a[N\times(N-1)/2,\ 0]$
$(A\times arctan(y)+Z)\uparrow(7+Q)$
if q **then** $n-1$ **else** n
if $a<0$ **then** A/B **else if** $b=0$ **then** B/A **else** z

the first arithmetic expression following this Boolean (the largest arithmetic expression found in this position is understood). The construction:

else ⟨simple arithmetic expression⟩

is equivalent to the construction:

else if true then ⟨simple arithmetic expression⟩

3.3.4. Operators and types

Apart from the Boolean expressions of if clauses, the constituents of simple arithmetic expressions must be of types **real** or **integer** (cf. section 5.1. Type Declarations). The meaning of the basic operators and the types of the expressions to which they lead are given by the following rules:

3.3.4.1. The operators +, −, and × have the conventional meaning (addition, subtraction, and multiplication). The type of the expression will be **integer** if both of the operands are of **integer** type, otherwise **real**.

3.3.4.2. The operations ⟨term⟩/⟨factor⟩ and ⟨term⟩ ÷ ⟨factor⟩ both denote division, to be understood as a multiplication of the term by the reciprocal of the factor with due regard to the rules of precedence (cf. section 3.3.5). Thus for example

$$a/b\times 7/(p-q)\times v/s$$

means

$$((((a\times(b^{-1}))\times 7)\times((p-q)^{-1}))\times v)\times(s^{-1})$$

The operator / is defined for all four combinations of types **real** and **integer** and will yield results of **real** type in any case. The operator ÷ is defined only for two operands both of type **integer** and will yield a result of type **integer**, mathematically defined as follows:

$$a\div b = sign\ (a/b)\times entier(abs(a/b))$$

(cf. sections 3.2.4 and 3.2.5).

3.3.4.3. The operation ⟨factor⟩↑⟨primary⟩ denotes exponentiation, where the factor is the base and the primary is the exponent. Thus, for example,

$$2\uparrow n\uparrow k \quad \text{means} \quad (2^n)^k$$

while

$$2\uparrow(n\uparrow m) \quad \text{means} \quad 2^{(n^m)}$$

Writing i for a number of **integer** type, r for a number of **real** type, and a for a number of either **integer** or **real** type, the result is given by the following rules:

$a\uparrow i$ If $i>0$, $a\times a\times\ \ldots\ \times a$ (i times), of the same type as a.
If $i=0$, if $a\neq 0$, 1, of the same type as a.
if $a=0$, undefined.
If $i<0$, if $a\neq 0$, $1/(a\times a\times\ \ldots\ \times a)$ (the denominator has $-i$ factors), of type **real**.
if $a=0$, undefined.

$a\uparrow r$ If $a>0$, $exp(r\times ln(a))$, of type **real**.
If $a=0$, if $r>0$, 0.0, of type **real**.
if $r\leqq 0$, undefined.
If $a<0$, always undefined.

3.3.5. Precedence of operators

The sequence of operations within one expression is generally from left to right, with the following additional rules:

3.3.5.1. According to the syntax given in section **3.3.1** the following rules of precedence hold:

first: $\uparrow$
second: $\times / \div$
third: $+ -$

3.3.5.2. The expression between a left parenthesis and the matching right parenthesis is evaluated by itself and this value is used in subsequent calculations. Consequently the desired order of execution of operations within an expression can always be arranged by appropriate positioning of parentheses.

3.3.6. Arithmetics of **real** quantities

Numbers and variables of type **real** must be interpreted in the sense of numerical analysis, i.e. as entities defined inherently with only a finite accuracy. Similarly, the possibility of the occurrence of a finite deviation from the mathematically defined result in any arithmetic expression is explicitly understood. No exact arithmetic will be specified, however, and it is indeed understood that different hardware representations may evaluate arithmetic expressions differently. The control of the possible consequences of such differences must be carried out by the methods of numerical analysis. This control must be considered a part of the process to be described, and will therefore be expressed in terms of the language itself.

3.4. Boolean Expressions

3.4.1. Syntax

⟨relational operator⟩ ::= $<$ | $\leq$ | $=$ | $\geq$ | $>$ | $\neq$
⟨relation⟩ ::= ⟨simple arithmetic expression⟩
  ⟨relational operator⟩⟨simple arithmetic expression⟩
⟨Boolean primary⟩ ::= ⟨logical value⟩|⟨variable⟩|
  ⟨function designator⟩|⟨relation⟩|(⟨Boolean expression⟩)
⟨Boolean secondary⟩ ::= ⟨Boolean primary⟩|$\neg$⟨Boolean primary⟩
⟨Boolean factor⟩ ::= ⟨Boolean secondary⟩|
  ⟨Boolean factor⟩$\wedge$⟨Boolean secondary⟩
⟨Boolean term⟩ ::= ⟨Boolean factor⟩|⟨Boolean term⟩
  $\vee$⟨Boolean factor⟩
⟨implication⟩ ::= ⟨Boolean term⟩|⟨implication⟩$\supset$⟨Boolean term⟩
⟨simple Boolean⟩ ::= ⟨implication⟩|
  ⟨simple Boolean⟩$\equiv$⟨implication⟩
⟨Boolean expression⟩ ::= ⟨simple Boolean⟩|
  ⟨if clause⟩⟨simple Boolean⟩ **else** ⟨Boolean expression⟩

3.4.2. Examples

$x = -2$
$Y > V \vee z < q$
$a+b > -5 \wedge z-d > q\uparrow 2$
$p \wedge q \vee x \neq y$
$g \equiv \neg a \wedge b \wedge \neg c \vee d \vee e \supset \neg f$
if $k<1$ **then** $s>w$ **else** $h \leq c$
if if if a **then** b **else** c **then** d **else** f **then** g **else** $h<k$

3.4.3. Semantics

A Boolean expression is a rule for computing a logical value. The principles of evaluation are entirely analogous to those given for arithmetic expressions in section 3.3.3.

3.4.4. Types

Variables and function designators entered as Boolean primaries must be declared **Boolean** (cf. section 5.1. Type Declarations and section **5.4.4.** Values of Function Designators).

3.4.5. The operators

Relations take on the value **true** whenever the corresponding relation is satisfied for the expressions involved, otherwise **false.**

The meaning of the logical operators $\neg$ (not), $\wedge$ (and), $\vee$ (or), $\supset$ (implies), and $\equiv$ (equivalent), is given by the following function table.

b1	**false**	**false**	**true**	**true**
b2	**false**	**true**	**false**	**true**
$\neg$b1	**true**	**true**	**false**	**false**
b1$\wedge$b2	**false**	**false**	**false**	**true**
b1$\vee$b2	**false**	**true**	**true**	**true**
b1$\supset$b2	**true**	**true**	**false**	**true**
b1$\equiv$b2	**true**	**false**	**false**	**true**

3.4.6. Precedence of operators

The sequence of operations within one expression is generally from left to right, with the following additional rules:

3.4.6.1. According to the syntax given in section 3.4.1 the following rules of precedence hold:

first: arithmetic expressions according to section 3.3.5.
second: $< \leq = \geq > \neq$
third: $\neg$
fourth: $\wedge$
fifth: $\vee$
sixth: $\supset$
seventh: $\equiv$

3.4.6.2. The use of parentheses will be interpreted in the sense given in section 3.3.5.2.

3.5. Designational Expressions

3.5.1. Syntax

⟨label⟩ ::= ⟨identifier⟩|⟨unsigned integer⟩
⟨switch identifier⟩ ::= ⟨identifier⟩
⟨switch designator⟩ ::= ⟨switch identifier⟩[⟨subscript expression⟩]
⟨simple designational expression⟩ ::= ⟨label⟩|⟨switch designator⟩|
  (⟨designational expression⟩)
⟨designational expression⟩ ::= ⟨simple designational expression⟩|
  ⟨if clause⟩⟨simple designational expression⟩ **else**
  ⟨designational expression⟩

3.5.2. Examples

17
$p9$
$Choose[n-1]$
$Town$[**if** $y<0$ **then** N **else** $N+1$]
if $Ab<c$ **then** 17 **else** q[**if** $w \leq 0$ **then** 2 **else** n]

3.5.3. Semantics

A designational expression is a rule for obtaining a label of a statement (cf. section 4. Statements). Again the principle of the evaluation is entirely analogous to that of arithmetic expressions (section 3.3.3). In the general case the Boolean expressions of the if clauses will select a simple designational expression. If this is a label the desired result is already found. A switch designator refers to the corresponding switch declaration (cf. section 5.3.

Switch Declarations) and by the actual numerical value of its subscript expression selects one of the designational expressions listed in the switch declaration by counting these from left to right. Since the designational expression thus selected may again be a switch designator this evaluation is obviously a recursive process.

3.5.4. The subscript expression

The evaluation of the subscript expression is analogous to that of subscripted variables (cf. section 3.1.4.2). The value of a switch designator is defined only if the subscript expression assumes one of the positive values 1, 2, 3, . . . , n, where n is the number of entries in the switch list.

3.5.5. Unsigned integers as labels

Unsigned integers used as labels have the property that leading zeros do not affect their meaning, e.g. 00217 denotes the same label as 217.

4. Statements

The units of operation within the language are called statements. They will normally be executed consecutively as written. However, this sequence of operations may be broken by go to statements, which define their successor explicitly, and shortened by conditional statements, which may cause certain statements to be skipped.

In order to make it possible to define a specific dynamic succession, statements may be provided with labels.

Since sequences of statements may be grouped together into compound statements and blocks the definition of statement must necessarily be recursive. Also since declarations, described in section 5, enter fundamentally into the syntactic structure, the syntactic definition of statements must suppose declarations to be already defined.

4.1. Compound Statements and Blocks

4.1.1. Syntax

⟨unlabelled basic statement⟩ ::= ⟨assignment statement⟩| ⟨go to statement⟩|⟨dummy statement⟩|⟨procedure statement⟩
⟨basic statement⟩ ::= ⟨unlabelled basic statement⟩|⟨label⟩: ⟨basic statement⟩
⟨unconditional statement⟩ ::= ⟨basic statement⟩| ⟨compound statement⟩|⟨block⟩
⟨statement⟩ ::= ⟨unconditional statement⟩| ⟨conditional statement⟩|⟨for statement⟩
⟨compound tail⟩ ::= ⟨statement⟩ **end** |⟨statement⟩ ; ⟨compound tail⟩
⟨block head⟩ ::= **begin**⟨declaration⟩|⟨block head⟩ ; ⟨declaration⟩
⟨unlabelled compound⟩ ::= **begin** ⟨compound tail⟩
⟨unlabelled block⟩ ::= ⟨block head⟩ ; ⟨compound tail⟩
⟨compound statement⟩ ::= ⟨unlabelled compound⟩| ⟨label⟩:⟨compound statement⟩
⟨block⟩ ::= ⟨unlabelled block⟩|⟨label⟩:⟨block⟩
⟨program⟩ ::= ⟨block⟩|⟨compound statement⟩

This syntax may be illustrated as follows: Denoting arbitrary statements, declarations, and labels, by the letters S, D, and L, respectively, the basic syntactic units take the forms:

Compound statement:

L: L: ... **begin** S ; S ; ... S ; S **end**

Block:

L: L: ... **begin** D ; D ; .. D ; S ; S ; ..S ; S **end**

It should be kept in mind that each of the statements S may again be a complete compound statement or block.

4.1.2. Examples

Basic statements:

```
a := p+q
go to Naples
START: CONTINUE: W := 7.993
```

Compound statement:

```
begin x := 0 ; for y := 1 step 1 until n do
        x := x+A[y] ;
    if x>q then go to STOP else if x>w−2 then
        go to S ;
    Aw: St: W := x+bob end
```

Block:

```
Q: begin integer i, k ; real w ;
    for i := 1 step 1 until m do
    for k := i+1 step 1 until m do
    begin w := A[i, k] ;
        A[i, k] := A[k, i] ;
        A[k, i] := w end for i and k
end block Q
```

4.1.3. Semantics

Every block automatically introduces a new level of nomenclature. This is realized as follows: Any identifier occurring within the block may through a suitable declaration (cf. section 5. Declarations) be specified to be local to the block in question. This means (a) that the entity represented by this identifier inside the block has no existence outside it, and (b) that any entity represented by this identifier outside the block is completely inaccessible inside the block.

Identifiers (except those representing labels) occurring within a block and not being declared to this block will be nonlocal to it, i.e. will represent the same entity inside the block and in the level immediately outside it. A label separated by a colon from a statement, i.e. labelling that statement, behaves as though declared in the head of the smallest embracing block, i.e. the smallest block whose brackets **begin** and **end** enclose that statement. In this context a procedure body must be considered as if it were enclosed by **begin** and **end** and treated as a block.

Since a statement of a block may again itself be a block the concepts local and nonlocal to a block must be understood recursively. Thus an identifier, which is nonlocal to a block A, may or may not be nonlocal to the block B in which A is one statement.

4.2. Assignment Statements

4.2.1. Syntax

⟨left part⟩ ::= ⟨variable⟩ := |⟨procedure identifier⟩ :=
⟨left part list⟩ ::= ⟨left part⟩|⟨left part list⟩⟨left part⟩
⟨assignment statement⟩ ::= ⟨left part list⟩⟨arithmetic expression⟩| ⟨left part list⟩⟨Boolean expression⟩

4.2.2. Examples

$s := p[0] := n := n+1+s$
$n := n+1$
$A := B/C-v-q\times S$
$S[v,k+2] := 3-arctan(s\times zeta)$
$V := Q>Y\wedge Z$

4.2.3. Semantics

Assignment statements serve for assigning the value of an expression to one or several variables or procedure identifiers. Assignment to a procedure identifier may only occur within the body of a procedure defining the value of a function designator (cf. section 5.4.4). The process will in the general case be understood to take place in three steps as follows:

4.2.3.1. Any subscript expressions occurring in the left part variables are evaluated in sequence from left to right.

4.2.3.2. The expression of the statement is evaluated.

4.2.3.3. The value of the expression is assigned to all the left part variables, with any subscript expressions having values as evaluated in step 4.2.3.1.

4.2.4. Types

The type associated with all variables and procedure identifiers of a left part list must be the same. If this type is **Boolean,** the expression must likewise be **Boolean.** If the type is **real** or **integer,** the expression must be arithmetic. If the type of the arithmetic expression differs from that associated with the variables and procedure identifiers, appropriate transfer functions are understood to be automatically invoked. For transfer from **real** to **integer** type, the transfer function is understood to yield a result equivalent to

$$entier(E+0.5)$$

where E is the value of the expression. The type associated with a procedure identifier is given by the declarator which appears as the first symbol of the corresponding procedure declaration (cf. section 5.4.4).

4.3. Go To Statements

4.3.1. Syntax

⟨go to statement⟩ ::= **go to** ⟨designational expression⟩

4.3.2. Examples

go to 8
go to *exit* [n+1]
go to *Town*[**if** $y<0$ **then** N **else** $N+1$]
go to if $Ab<c$ **then** 17 **else** q[**if** $w<0$ **then** 2 **else** n]

4.3.3. Semantics

A go to statement interrupts the normal sequence of operations, defined by the write-up of statements, by defining its successor explicitly by the value of a designational expression. Thus the next statement to be executed will be the one having this value as its label.

4.3.4. Restriction

Since labels are inherently local, no go to statement can lead from outside into a block. A go to statement may, however, lead from outside into a compound statement.

4.3.5. Go to an undefined switch designator

A go to statement is equivalent to a dummy statement if the designational expression is a switch designator whose value is undefined.

4.4. Dummy Statements

4.4.1. Syntax

⟨dummy statement⟩ ::= ⟨empty⟩

4.4.2. Examples

L:
begin ... ; *John*: **end**

4.4.3. Semantics

A dummy statement executes no operation. It may serve to place a label.

4.5. Conditional Statements

4.5.1. Syntax

⟨if clause⟩ ::= **if** ⟨Boolean expression⟩ **then**
⟨unconditional statement⟩ ::= ⟨basic statement⟩|
⟨compound statement⟩|⟨block⟩
⟨if statement⟩ ::= ⟨if clause⟩ ⟨unconditional statement⟩
⟨conditional statement⟩ ::= ⟨if statement⟩|⟨if statement⟩ **else**
⟨statement⟩|⟨if clause⟩⟨for statement⟩|
⟨label⟩ : ⟨conditional statement⟩

4.5.2. Examples

if $x>0$ **then** $n := n+1$
if $v>u$ **then** V: $q := n+m$ **else go to** R
if $s<0\vee P\leqq Q$ **then** AA: **begin if** $q<v$ **then** $a := v/s$
else $y := 2\times a$ **end**
else if $v>s$ **then** $a := v-q$ **else if** $v>s-1$
then go to S

4.5.3. Semantics

Conditional statements cause certain statements to be executed or skipped depending on the running values of specified Boolean expressions.

4.5.3.1. If statement. The unconditional statement of an if statement will be executed if the Boolean expression of the if clause is true. Otherwise it will be skipped and the operation will be continued with the next statement.

4.5.3.2. Conditional statement. According to the syntax two different forms of conditional statements are possible. These may be illustrated as follows:

if B1 **then** S1 **else if** B2 **then** S2 **else** S3 ; S4

and

if B1 **then** S1 **else if** B2 **then** S2 **else if** B3 **then** S3 ; S4

Here B1 to B3 are Boolean expressions, while S1 to S3 are unconditional statements. S4 is the statement following the complete conditional statement.

The execution of a conditional statement may be described as follows: The Boolean expression of the if clauses are evaluated one after the other in sequence from left to right until one yielding the value **true** is found. Then the unconditional statement following this Boolean is executed. Unless this statement defines its successor explicitly the next statement to be executed will be S4, i.e. the statement following the complete conditional statement. Thus the effect of the delimiter **else** may be described by saying that it defines the successor of the statement it follows to be the statement following the complete conditional statement.

The construction

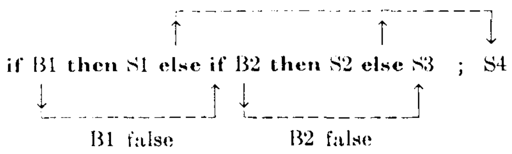
else ⟨unconditional statement⟩

is equivalent to

else if true then ⟨unconditional statement⟩

If none of the Boolean expressions of the if clauses is true, the effect of the whole conditional statement will be equivalent to that of a dummy statement.

For further explanation the following picture may be useful:



4.5.4. Go to into a conditional statement

The effect of a go to statement leading into a conditional statement follows directly from the above explanation of the effect of **else**.

4.6. FOR STATEMENTS

4.6.1. Syntax

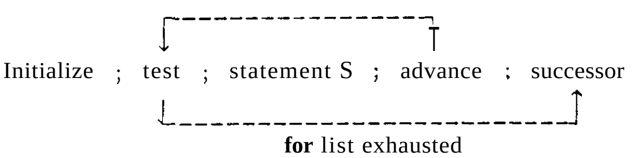
⟨for list element⟩ ::= ⟨arithmetic expression⟩|
⟨arithmetic expression⟩ **step** ⟨arithmetic expression⟩ **until** ⟨arithmetic expression⟩|⟨arithmetic expression⟩ **while** ⟨Boolean expression⟩
⟨for list⟩ ::= ⟨for list element⟩|⟨for list⟩ , ⟨for list element⟩
⟨for clause⟩ ::= **for** ⟨variable⟩ := ⟨for list⟩ **do**
⟨for statement⟩ ::= ⟨for clause⟩⟨statement⟩|
⟨label⟩: ⟨for statement⟩

4.6.2. Examples

for q := 1 **step** s **until** n **do** $A[q]$:= $B[q]$
for k := 1, $V1\times 2$ **while** $V1<N$ **do**
    for j := $I+G$, L, 1 **step** 1 **until** N, $C+D$ **do**
        $A[k,j]$:= $B[k,j]$

4.6.3. Semantics

A for clause causes the statement S which it precedes to be repeatedly executed zero or more times. In addition it performs a sequence of assignments to its controlled variable. The process may be visualized by means of the following picture:



In this picture the word initialize means: perform the first assignment of the for clause. Advance means: perform the next assignment of the for clause. Test determines if the last assignment has been done. If so, the execution continues with the successor of the for statement. If not, the statement following the for clause is executed.

4.6.4. The for list elements

The for list gives a rule for obtaining the values which are consecutively assigned to the controlled variable. This sequence of values is obtained from the for list elements by taking these one by one in the order in which they are written. The sequence of values generated by each of the three species of for list elements and the corresponding execution of the statement S are given by the following rules:

4.6.4.1. Arithmetic expression. This element gives rise to one value, namely the value of the given arithmetic expression as calculated immediately before the corresponding execution of the statement S.

4.6.4.2. Step-until-element. An element of the form A **step** B **until** C, where A, B, and C, are arithmetic expressions, gives rise to an execution which may be described most concisely in terms of additional ALGOL statements as follows:

    V := A ;
L1: **if** $(V-C)\times sign(B)>0$ **then go to** *element exhausted*;
    statement S ;
    V := V+B ;
    go to *L1* ;

where V is the controlled variable of the for clause and *element exhausted* points to the evaluation according to the next element in the for list, or if the step-until-element is the last of the list, to the next statement in the program.

4.6.4.3. While-element. The execution governed by a for list element of the form E **while** F, where E is an arithmetic and F a Boolean expression, is most concisely described in terms of additional ALGOL statements as follows:

L3: V := E ;
    if ¬F **then go to** *element exhausted* ;
    Statement S ;
    go to *L3* ;

where the notation is the same as in 4.6.4.2 above.

4.6.5. The value of the controlled variable upon exit

Upon exit out of the statement S (supposed to be compound) through a go to statement the value of the controlled variable will be the same as it was immediately preceding the execution of the go to statement.

If the exit is due to exhaustion of the for list, on the other hand, the value of the controlled variable is undefined after the exit.

4.6.6. Go to leading into a for statement

The effect of a go to statement, outside a for statement, which refers to a label within the for statement, is undefined.

4.7. PROCEDURE STATEMENTS

4.7.1. Syntax

⟨actual parameter⟩ ::= ⟨string⟩|⟨expression⟩|⟨array identifier⟩|
⟨switch identifier⟩|⟨procedure identifier⟩
⟨letter string⟩ ::= ⟨letter⟩|⟨letter string⟩⟨letter⟩

⟨parameter delimiter⟩ ::= ,|)⟨letter string⟩:(
⟨actual parameter list⟩ ::= ⟨actual parameter⟩|
    ⟨actual parameter list⟩⟨parameter delimiter⟩
    ⟨actual parameter⟩
⟨actual parameter part⟩ ::= ⟨empty⟩|
    (⟨actual parameter list⟩)
⟨procedure statement⟩ ::= ⟨procedure identifier⟩
    ⟨actual parameter part⟩

4.7.2. Examples

Spur (*A*)Order: (7)Result to: (*V*)
Transpose (*W*,*v*+1)
Absmax(*A*,*N*,*M*,*Yy*,*I*,*K*)
Innerproduct(*A*[*t*,*P*,*u*],*B*[*P*],10,*P*,*Y*)

These examples correspond to examples given in section **5.4.2.**

4.7.3. Semantics

A procedure statement serves to invoke (call for) the execution of a procedure body (cf. section **5.4.** Procedure Declarations). Where the procedure body is a statement written in ALGOL the effect of this execution will be equivalent to the effect of performing the following operations on the program at the time of execution of the procedure statement:

4.7.3.1. Value assignment (call by value)

All formal parameters quoted in the value part of the procedure declaration heading are assigned the values (cf. section 2.8. Values and Types) of the corresponding actual parameters, these assignments being considered as being performed explicitly before entering the procedure body. The effect is as though an additional block embracing the procedure body were created in which these assignments were made to variables local to this fictitious block with types as given in the corresponding specifications (cf. section 5.4.5). As a consequence, variables called by value are to be considered as nonlocal to the body of the procedure, but local to the fictitious block (cf. section 5.4.3).

4.7.3.2. Name replacement (call by name)

Any formal parameter not quoted in the value list is replaced, throughout the procedure body, by the corresponding actual parameter, after enclosing this latter in parentheses wherever syntactically possible. Possible conflicts between identifiers inserted through this process and other identifiers already present within the procedure body will be avoided by suitable systematic changes of the formal or local identifiers involved.

4.7.3.3. Body replacement and execution

Finally the procedure body, modified as above, is inserted in place of the procedure statement and executed. If the procedure is called from a place outside the scope of any nonlocal quantity of the procedure body the conflicts between the identifiers inserted through this process of body replacement and the identifiers whose declarations are valid at the place of the procedure statement or function designator will be avoided through suitable systematic changes of the latter identifiers.

4.7.4. Actual-formal correspondence

The correspondence between the actual parameters of the procedure statement and the formal parameters of the procedure heading is established as follows: The actual parameter list of the procedure statement must have the same number of entries as the formal parameter list of the procedure declaration heading. The correspondence is obtained by taking the entries of these two lists in the same order.

4.7.5. Restrictions

For a procedure statement to be defined it is evidently necessary that the operations on the procedure body defined in sections **4.7.3.1** and **4.7.3.2** lead to a correct ALGOL statement.

This imposes the restriction on any procedure statement that the kind and type of each actual parameter be compatible with the kind and type of the corresponding formal parameter. Some important particular cases of this general rule are the following:

4.7.5.1. If a string is supplied as an actual parameter in a procedure statement or function designator, whose defining procedure body is an ALGOL 60 statement (as opposed to non-ALGOL code, cf. section **4.7.8**), then this string can only be used within the procedure body as an actual parameter in further procedure calls. Ultimately it can only be used by a procedure body expressed in non-ALGOL code.

4.7.5.2. A formal parameter which occurs as a left part variable in an assignment statement within the procedure body and which is not called by value can only correspond to an actual parameter which is a variable (special case of expression).

4.7.5.3. A formal parameter which is used within the procedure body as an array identifier can only correspond to an actual parameter which is an array identifier of an array of the same dimensions. In addition if the formal parameter is called by value the local array created during the call will have the same subscript bounds as the actual array.

4.7.5.4. A formal parameter which is called by value cannot in general correspond to a switch identifier or a procedure identifier or a string, because these latter do not possess values (the exception is the procedure identifier of a procedure declaration which has an empty formal parameter part, (cf. section 5.4.1) and which defines the value of a function designator (cf. section 5.4.4). This procedure identifier is in itself a complete expression).

4.7.5.5. Any formal parameter may have restrictions on the type of the corresponding actual parameter associated with it (these restrictions may, or may not, be given through specifications in the procedure heading). In the procedure statement such restrictions must evidently be observed.

4.7.6. Deleted.

4.7.7. Parameter delimiters

All parameter delimiters are understood to be equivalent. No correspondence between the parameter delimiters used in a procedure statement and those used in the procedure heading is expected beyond their number being the i.e. Thus the information conveyed by using the elaborate ones is entirely optional.

4.7.8. Procedure body expressed in code

The restrictions imposed on a procedure statement calling a procedure having its body expressed in non-ALGOL code evidently can only be derived from the characteristics of the code used and the intent of the user and thus fall outside the scope of the reference language.

Declarations

Declarations serve to define certain properties of the quantities used in the program, and to associate them with identifiers. A declaration of an identifier is valid for one block. Outside this block the particular identifier may be used for other purposes (cf. section 4.1.3).

Dynamically this implies the following: at the time of an entry into a block (through the **begin**, since the labels inside are local and therefore inaccessible from outside) all identifiers declared for the block assume the significance implied by the nature of the declarations given. If these identifiers had already been defined by other declarations outside they are for the time being given a new significance. Identifiers which are not declared for the block, on the other hand, retain their old meaning.

At the time of an exit from a block (through **end**, or by a go to statement) all identifiers which are declared for the block lose their local significance.

A declaration may be marked with the additional declarator **own**. This has the following effect: upon a re-entry into the block, the values of own quantities will be unchanged from their values at the last exit, while the values of declared variables which are not marked as own are undefined. Apart from labels and formal parameters of procedure declarations and with the possible exception of those for standard functions (cf. sections 3.2.4 and 3.2.5), all identifiers of a program must be declared. No identifier may be declared more than once in any one block head.

Syntax.

⟨declaration⟩ ::= ⟨type declaration⟩|⟨array declaration⟩|
  ⟨switch declaration⟩|⟨procedure declaration⟩

5.1. Type Declarations

5.1.1. Syntax

⟨type list⟩ ::= ⟨simple variable⟩|
  ⟨simple variable⟩, ⟨type list⟩
⟨type⟩ ::= **real** | **integer** | **Boolean**
⟨local or own type⟩ ::= ⟨type⟩|**own** ⟨type⟩
⟨type declaration⟩ ::= ⟨local or own type⟩⟨type list⟩

5.1.2. Examples

integer p,q,s
own Boolean $Acryl,n$

5.1.3. Semantics

Type declarations serve to declare certain identifiers to represent simple variables of a given type. Real declared variables may only assume positive or negative values including zero. Integer declared variables may only assume positive and negative integral values including zero. Boolean declared variables may only assume the values **true** and **false**.

In arithmetic expressions any position which can be occupied by a real declared variable may be occupied by an integer declared variable.

For the semantics of **own**, see the fourth paragraph of section 5 above.

5.2. Array Declarations

5.2.1. Syntax

⟨lower bound⟩ ::= ⟨arithmetic expression⟩
⟨upper bound⟩ ::= ⟨arithmetic expression⟩
⟨bound pair⟩ ::= ⟨lower bound⟩:⟨upper bound⟩
⟨bound pair list⟩ ::= ⟨bound pair⟩|⟨bound pair list⟩,⟨bound pair⟩
⟨array segment⟩ ::= ⟨array identifier⟩[⟨bound pair list⟩]|
  ⟨array identifier⟩,⟨array segment⟩
⟨array list⟩ ::= ⟨array segment⟩|⟨array list⟩,⟨array segment⟩
⟨array declaration⟩ ::= **array** ⟨array list⟩|⟨local or own type⟩
  array ⟨array list⟩

5.2.2. Examples

array $a, b, c[7:n,2:m], s[-2:10]$
own integer array $A[$**if** $c<0$ **then** 2 **else** $1:20]$
real array $q[-7:-1]$

5.2.3. Semantics

An array declaration declares one or several identifiers to represent multidimensional arrays of subscripted variables and gives the dimensions of the arrays, the bounds of the subscripts and the types of the variables.

5.2.3.1. Subscript bounds. The subscript bounds for any array are given in the first subscript bracket following the identifier of this array in the form of a bound pair list. Each item of this list gives the lower and upper bound of a subscript in the form of two arithmetic expressions separated by the delimiter : The bound pair list gives the bounds of all subscripts taken in order from left to right.

5.2.3.2. Dimensions. The dimensions are given as the number of entries in the bound pair lists.

5.2.3.3. Types. All arrays declared in one declaration are of the same quoted type. If no type declarator is given the type **real** is understood.

5.2.4. Lower upper bound expressions

5.2.4.1 The expressions will be evaluated in the same way as subscript expressions (cf. section 3.1.4.2).

5.2.4.2. The expressions can only depend on variables and procedures which are nonlocal to the block for which the array declaration is valid. Consequently in the outermost block of a program only array declarations with constant bounds may be declared.

5.2.4.3. An array is defined only when the values of all upper subscript bounds are not smaller than those of the corresponding lower bounds.

5.2.4.4. The expressions will be evaluated once at each entrance into the block.

5.2.5. The identity of subscripted variables

The identity of a subscripted variable is not related to the subscript bounds given in the array declaration. However, even if an array is declared **own** the values of the corresponding subscripted variables will, at any time, be defined only for those of these variables which have subscripts within the most recently calculated subscript bounds.

5.3. Switch Declarations

5.3.1. Syntax

⟨switch list⟩ ::= ⟨designational expression⟩|
⟨switch list⟩, ⟨designational expression⟩
⟨switch declaration⟩ ::= **switch** ⟨switch identifier⟩:= ⟨switch list⟩

5.3.2. Examples

```
switch S := S1,S2,Q[m], if v>-5 then S3 else S4
switch Q := p1,w
```

5.3.3. Semantics

A switch declaration defines the set of values of the corresponding switch designators. These values are given one by one as the values of the designational expressions entered in the switch list. With each of these designational expressions there is associated a positive integer, 1, 2, ... , obtained by counting the items in the list from left to right. The value of the switch designator corresponding to a given value of the subscript cxpression (cf. section 3.5. Designational Expressions) is the value of the designational expression in the switch list having this given value as its associated integer.

5.3.4. Evaluation of expressions in the switch list

An expression in the switch list will be evaluated every time the item of the list in which the expression occurs is referred to, using the current values of all variables involved.

5.3.5. Influence of scopes

If a switch designator occurs outside the scope of a quantity entering into a designational expression in the switch list, and an evaluation of this switch designator selects this designational expression, then the conflicts between the identifiers for the quantities in this expression and the identifiers whose declarations are valid at the place of the switch designator will be avoided through suitable systematic changes of the latter identifiers.

5.4. Procedure Declarations

5.4.1. Syntax

⟨formal parameter⟩ ::= ⟨identifier⟩
⟨formal parameter list⟩ ::= ⟨formal parameter⟩|
⟨formal parameter list⟩⟨parameter delimiter⟩
⟨formal parameter⟩
⟨formal parameter part⟩ ::= ⟨empty⟩|(⟨formal parameter list⟩)
⟨identifier list⟩ ::= ⟨identifier⟩|⟨identifier list⟩, ⟨identifier⟩
⟨value part⟩ ::= **value**⟨identifier list⟩ ; |⟨empty⟩
⟨specifier⟩ ::= **string**|⟨type⟩|**array**|⟨type⟩**array**|**label**|**switch**|
procedure|⟨type⟩**procedure**
⟨specification part⟩ ::= ⟨empty⟩|⟨specifier⟩⟨identifier list⟩ ; |
⟨specification part⟩⟨specifier⟩⟨identifier list⟩ ;
⟨procedure heading⟩ ::= ⟨procedure identifier⟩
⟨formal parameter part⟩ ; ⟨value part⟩⟨specification part⟩
⟨procedure body⟩ ::= ⟨statement⟩|⟨code⟩
⟨procedure declaration⟩ ::=
procedure ⟨procedure heading⟩⟨procedure body⟩|
⟨type⟩ **procedure** ⟨procedure heading⟩⟨procedure body⟩

5.4.2. Examples (see also the examples at the end o[f] the report)

```
procedure Spur(a)Order:(n)Result:(s) ; value n ;
array a ; integer n ; real s ;
begin integer k ;
s := 0 ;
for k := 1 step 1 until n do s := s+a[k,k]
end

procedure Transpose(a)Order:(n) ; value n ;
array a ; integer n ;
begin real w ; integer i, k ;
for i := 1 step 1 until n do
      for k := 1+i step 1 until n do
      begin w := a[i,k] ;
            a[i,k] := a[k,i] ;
            a[k,i] := w
      end
end Transpose

integer procedure Step (u) ; real u ;
Step := if 0≦u∧u≦1 then 1 else 0

procedure Absmax(a)size:(n,m)Result:(y)Subscripts:(i,k) ;
comment The absolute greatest element of the matrix a
   of size n by m is transferred to y, and the subscripts of th[e]
   element to i and k ;
array a ; integer n, m, i, k ; real y ;
begin integer p, q ;
y := 0 ;
for p := 1 step 1 until n do for q := 1 step 1 until m do
if abs(a[p,q])>y then begin y := abs(a[p,q]) ; i := p ;
  k := q
end end Absmax

procedure Innerproduct(a,b)Order:(k,p)Result:(y) ; value k ;
integer k,p ; real y,a,b ;
begin real s ;
s := 0 ;
for p := 1 step 1 until k do s := s+a×b ;
y := s
end Innerproduct
```

5.4.3. Semantics

A procedure declaration serves to define the procedu[re] associated with a procedure identifier. The principal co[n]stituent of a procedure declaration is a statement or [a] piece of code, the procedure body, which through the u[se] of procedure statements and/or function designators ma[y] be activated from other parts of the block in the head o[f] which the procedure declaration appears. Associated wit[h] the body is a heading, which specifies certain identifie[rs] occurring within the body to represent formal parameter[s]. Formal parameters in the procedure body will, wheneve[r] the procedure is activated (cf. section 3.2. Functio[n] Designators and section 4.7. Procedure Statement[s]) be assigned the values of or replaced by actual parameter[s]. Identifiers in the procedure body which are not form[al] will be either local or nonlocal to the body depending o[n] whether they are declared within the body or not. Tho[se] of them which are nonlocal to the body may well be loc[al] to the block in the head of which the procedure declar[a]tion appears. The procedure body always acts like ock, whether it has the form of one or not. Consequently ie scope of any label labelling a statement within the ody or the body itself can never extend beyond the pro-edure body. In addition, if the identifier of a formal arameter is declared anew within the procedure body ncluding the case of its use as a label as in section 4.1.3), is thereby given a local significance and actual param-ers which correspond to it are inaccessible throughout ie scope of this inner local quantity.

5.4.4. Values of function designators

For a procedure declaration to define the value of a inction designator there must, within the procedure ody, occur one or more explicit assignment statements ith the procedure identifier in a left part; at least one of iese must be executed, and the type associated with the rocedure identifier must be declared through the appear-nce of a type declarator as the very first symbol of the rocedure declaration. The last value so assigned is used continue the evaluation of the expression in which the inction designator occurs. Any occurrence of the pro-edure identifier within the body of the procedure other han in a left part in an assignment statement denotes ctivation of the procedure.

5.4.5. Specifications

In the heading a specification part, giving information bout the kinds and types of the formal parameters by ieans of an obvious notation, may be included. In this art no formal parameter may occur more than once. pecifications of formal parameters called by value (cf. ection 4.7.3.1) must be supplied and specifications of ormal parameters called by name (cf. section 4.7.3.2) iay be omitted.

5.4.6. Code as procedure body

It is understood that the procedure body may be ex-ressed in non-ALGOL language. Since it is intended that he use of this feature should be entirely a question of ardware representation, no further rules concerning iis code language can be given within the reference inguage

Examples of Procedure Declarations:

XAMPLE 1.

```
rocedure euler (fct, sum, eps, tim)   ;  value eps, tim   ;
iteger tim   ;  real procedure fct   ;  real sum, eps   ;
omment euler computes the sum of fct(i) for i from zero up to
ifinity by means of a suitabley refined euler transformation. The
immation is stopped as soon as tim times in succession the abso-
ite value of the terms of the transformed series are found to be
ess than eps. Hence, one should provide a function fct with one
iteger argument, an upper bound eps, and an integer tim. The
utput is the sum sum. euler is particularly efficient in the case
f a slowly convergent or divergent alternating series   ;
egin integer i, k, n, t   ;  array m[0:15]   ;  real mn, mp, ds   ;
:= n := t := 0   ;  m[0] := fct(0)   ;  sum := m[0]/2   ;
extterm: i := i+1   ;  mn := fct(i)   ;
        for k := 0 step 1 until n do
            begin mp := (mn+m[k])/2   ;  m[k] := mn   ;
              mn := mp end means   ;
        if (abs(mn)<abs(m[n]))∧(n<15) then
            begin ds := mn/2   ;  n := n+1   ;  m[n] :=
              mn end accept
        else ds := mn   ;
        sum := sum + ds   ;
        if abs(ds)<eps then t := t+1 else t := 0   ;
        if t<tim then go to nextterm
end euler
```

EXAMPLE 2.[8]

```
procedure RK(x,y,n,FKT,eps,eta,xE,yE,fi)   ;  value x,y   ;
integer n   ;  Boolean fi   ;  real x,eps,eta,xE   ;  array
y,yE   ;  procedure FKT   ;
comment: RK integrates the system y_k'=f_k(x,y_1,y_2,...,y_n)
(k=1,2,...,n) of differential equations with the method of Runge-
Kutta with automatic search for appropriate length of integration
step. Parameters are: The initial values x and y[k] for x and the un-
known functions y_k(x). The order n of the system. The procedure
FKT(x,y,n,z) which represents the system to be integrated, i.e.
the set of functions f_k . The tolerance values eps and eta which
govern the accuracy of the numerical integration. The end of the
integration interval xE. The output parameter yE which repre-
sents the solution at x=xE. The Boolean variable fi, which must
always be given the value true for an isolated or first entry into
RK. If however the functions y must be available at several mesh-
points x_0 , x_1 , ... , x_n , then the procedure must be called repeat-
edly (with x=x_k , xE=x_{k+1}, for k=0, 1, ... , n-1) and then the
later calls may occur with fi=false which saves computing time.
The input parameters of FKT must be x,y,n, the output parameter
z represents the set of derivatives z[k]=f_k(x,y[1], y[2], ... , y[n])
for x and the actual y's. A procedure comp enters as a nonlocal
identifier   ;
begin
  array z,y1,y2,y3[1:n]   ;  real x1,x2,x3,H   ;  Boolean out   ;
  integer k,j   ;  own real s,Hs   ;
  procedure RK1ST(x,y,h,xe,ye)   ;  real x,h,xe   ;  array
        y,ye   ;
    comment: RK1ST integrates one single RUNGE-KUTTA
            with initial values x,y[k] which yields the output
            parameters xe=x+h and ye[k], the latter being the
            solution at xe. Important: the parameters n, FKT, z
            enter RK1ST as nonlocal entities   ;
    begin
      array w[1:n], a[1:5]   ;  integer k,j   ;
      a[1] := a[2] := a[5] := h/2   ;  a[3] := a[4] := h   ;
      xe := x   ;
      for k := 1 step 1 until n do ye[k] := w[k] := y[k]   ;
      for j := 1 step 1 until 4 do
      begin
        FKT(xe,w,n,z)   ;
        xe := x+a[j]   ;
        for k := 1 step 1 until n do
        begin
          w[k] := y[k]+a[j]×z[k]   ;
          ye[k] := ye[k] + a[j+1]×z[k]/3
```

[8] This RK-program contains some new ideas which are related to ideas of S. GILL, A process for the step-by-step integration of differential equations in an automatic computing machine, [*Proc. Camb. Phil. Soc.* 47 (1951), 96]; and E. FRÖBERG, On the solution of ordinary differential equations with digital computing machines, [*Fysiograf. Sällsk. Lund, Förhd.* 20, 11 (1950), 136–1521. It must be clear, however, that with respect to computing time and round-off errors it may not be optimal, nos has it actually been tested on a computer.

```
        end k
      end j
    end RR1ST   ;
Begin of program:
    if fi then begin H := xE-x   ;  s := 0 end else H := Hs   ;
    out := false   ;
AA: if (x+2.01×H-xE>0)≡(H>0) then
    begin Hs := H   ; out := true   ; H := (xE-x)/2
      end if   ;
    RK1ST (x,y,2×H,x1,y1)   ;
BB: RK1ST (x,y,H,x2,y2)   ; RK1ST(x2,y2,H,x3,y3)   ;
    for k := 1 step 1 until n do
        if comp(y1[k],y3[k],eta)>eps then go to CC   ;
    comment: comp(a,bc,) is a function designator, the value
      of which is the absolute value of the difference of the
      mantissae of a and b, after the exponents of these quan-
      tities have been made equal to the largest of the exponent:;
      of the originally given parameters a,b,c   ;
    x := 23   ; if out then go to DD   ;
    for k := 1 step 1 until n do y[k] := y3[k]   ;
    if s=5 then begin s := 0   ; H := 2XH end if   ;
    s := s+1   ; go to AA   ;
CC: H := 0.5×H   ; out := false   ; x1 := x2   ;
    for k := 1 step 1 until n do y1[k] := y2[k]   ;
    go to BB   ;
DD: for k := 1 step 1 until n do yE[k] := y3[k]
end RK •
```

ALPHABETIC INDEX OF DEFINITIONS OF CONCEPTS AND SYNTACTIC UNITS

All references are given through section numbers. The references are given in three groups:

def Following the abbreviation "def", reference to the syntactic definition (if any) is given.

synt Following the abbreviation "synt", references to the occurrences in metalinguistic formulae are given. References already quoted in the def-group are not repeated.

text Following the word "text", the references to definitions given in the text are given.

The basic symbols represented by signs other than underlined words [in typewritten copy; boldface in printed copy—Ed.] have been collected at the beginning.

The examples have been ignored in compiling the index.

END OF THE REPORT

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