WARPIAN:
A SYSTEM FOR GENERATING PIANS.
by
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#### Abstract

The system is intended to be a general purpose plan generator for domains described in a formalism close to that of STRIPS. The method used is in many ways similar to STRIFS, but the search space is complete, unlike STRIPS and several similar systems. However the present purely depth-first search strategy is obviously incomplete although it produces good solutions to many problems.

The system is implemented in PROLOG, an elegant programming language essentially identical in syntax and semantics to Predicate Calculue in clausal form. The entire WARPLAN program comprises 46 clavses and ains at conciseness and clarity rather than efficiency. Nevertheless the present implementation solves some "standard" problems roughly 8 times faster than STRIPS, or rough1y 5 times slower than LAWALY (a system designed primarily to be efficient).

WARPLAN is perhaps interesting as a system implemented in First Order Logic which solves problems in First Order Logic.


## pREFACE

This menc is an interim report on a program I implenented and tested during the last two weeks of a visit to the University of Marseille. I am obviously hoping to develop the program further, but as a number of people have expressed interest in the work, it seems worthwhile to present the basic ideas now.

The most important part of this memo is Appendix $I$.

## 1. INTRODUCTION

Many problem domains can naturally be formalised as a world with a set of actions which transform that world from one state to another. A particular problen is then specified by describing an initial state and a desired goal state. The problem solver is required to generate a plan, a simple sequence of actions which transforms the world from the initial state to the goal state (strictly speaking, from any state satisfying the initial description to some state satisfying the goal description).

Such problem domains include, but are not restricted to, applications to robot planning. More generally the problem can be regarded as one of compiling a high level goal description into a low level program, albeit of an extremely simple structure, given a formal description of the target language (ie. target machine, ie. "world").

Writing special purpose plan generators (for a particular world or target machine) is at best tedious, and at worst near impossible, if the specification of the world is liable to change. A number of systems (including $\operatorname{STRIPS},^{\{3\}} \operatorname{LAWALY},{ }^{\{14\}} \operatorname{HACKER}^{\{16\}}$ ) have been implemented which attempt in varying degrees to be general, ie. applicable to any domain.

A common failing of present systems is that they require that a conjunction of goals can be solved under the linear assumption. That is, goals ' $X_{1}{ }_{\&}{ }_{\&} X_{2}{ }^{\prime}$ can be solved starting from some state 'start' by a plan of the form 'start; $T_{1} ; T_{2}^{\prime}$ where $T_{1}$ is the action sequence of a minimal* plan to achieve $X_{1}$ from 'start' and $T_{2}$ is the sequence of a minimal plan to achieve $X_{2}$ from the state resulting from the plan 'start; $T_{1}{ }^{\prime}$. If ${ }^{\prime} X_{1} \mathcal{R}_{1} X_{2}^{\prime}$ is unsolvable in that order, the goals are typically permuted to ' $X_{2}{ }^{2}: X_{1}^{\prime}$ ' and another attempt is made. There is no facility to interleave subplans. This frequently means that an optimal solution is not even theoretically attainable - it is not in the search space. At worst, there is no solution in the search space even though there is a solution in the intended interpretation (see the 3 Blocks Problem below). We shall call such systems linear,

WARPLAN can be regarded as a simple extension to STRIPS sufficient to attain completeness. The extension obviates the need to permute goals in a conjunction. The extension is also irredundant, that is the identical plan cannot be generated in two different ways. However there is still the underlying redundancy that re-ordering independent actions produces a distinct plan.

[^0]2. THE 3 BLOCKS PROBLEM (of Austin Tate)

This is possibly the simplest example of a problem which is not solvable (optimally) by a linear planning systems. It was first noted as such by Austin Tate, although ir appears in Sussman's thesis ${ }^{\{16\}}$ as an "anomalous situation".

Given an initial state of 3 blocks as shown below, we want to achieve a final state in which ' $a$ ' is on ' $b$ ' and ' $b$ ' is on ' $c$ ' :-


Typically, a STRIPS-1ike system would go through the following steps:-

: FJRST GOAL ACHIEVED


The system is now trying to clear the top of ' $b$ ' so that it can put ' $b$ ' on 'c'. But this would destroy an already achieved subgoal: 'on (a,b)'. Typically, the system tries again with permuted subgoals:-

## 5



PROTECTION VIOLATION

The system is trying to clear the top of 'a' so that it can put 'a' on 'b', and to do this it first needs to remove 'b' - another protection violation, this time of 'on(b, c)'.

The system is now in a dilemma since all paths in its search space lead to a protection violation. It either concludes, falsely, that the problem is unsolvable, or else permits a protection violation which (for this problem) allows a non-optimal solution to be obtained.

The optimal solution is of course:-

```
move c from a to floor;
```

move $b$ from floor to $c$;
move $a$ from floor to $b$
The reason linear planning systems get into difficulties is that the solutions to the two main goals are interleaved in the optimal plan. ie. the first and third steps achieve 'on(a,b)' and the second achieves 'on(b,c)'.

Note: Refer to Appendix III for details of PROIOG syntax.
All data items manipulated by WARPLAN are represented as PC (Predicate Calculus, First Order Logic) terms. The main data types are a conjunction of facts (or goals) and a ${ }^{1}$ an. A conjunction is constructed from certain primitive data items called facts using the binary function '\&'. Similarly a plan is constructed from primitive data items cailed initial states and actions using the binary function ';'. '\&' and ';' are declared as infix operators so that:-

$$
\begin{aligned}
& X \& Y \& Z=X \&(Y \& Z)=\&_{1}(X, \&(Y, Z)) \\
& T ; U ; V=(T ; U) ; V=;(;(T, U), V)
\end{aligned}
$$

PROLOG treats the above identities as different external representations of the same internal object.
Interpret: ' $X \& Y^{\prime}$. as $!X$ and $Y$ '
and ' $T$; $U$ ' as 'the state after doing $U$ in $T$ '.
Note that, for brevity of exposition, we will identify a conjunction of a single fact with that fact and will frequently not distinguish between a plan and the state resulting from that plan.

A problem domain is specified to WARPLAN as a set of clauses, the problem database. This contains essentially the same information as STRIPS add-1ists, delete-lists, preconditions and initial world wff. Certain information in the problem database may be represented procedurally, ie. as non-unit clauses. Usually, however, the clauses will be unit assertions. The following predicates. are used:-
$+\operatorname{add}(X, U) \quad:$ fact $X$ is added by action $U$; ie. $X$ is true in any state resulting from $U$ (and $U$ is a possible action in some state in which $X$ is not true).
$+\operatorname{del}(X, U) \quad:$ fact $X$ is "deleted" by action $U$; ie. it is not the case that $X$ is preserved by $U$. ( $X$ is preserved by $U$ if and only if $X$ is not added by $U$ and $X$ is true in a state resulting from $U$ whenever $X$ is true in the preceding state.) .

```
+ can(U,C) : the conjunction of facts C is che preconditions of
    action U; ie. U is possible in any state in which
    C is true.
+ always(X) : fact }X\mathrm{ is true in any state.
+ imposs(C) : the conjuncrion of facts C is impossible in any state.
+ given(T,X) : fact X is true in the initiai state T (but it is not
    the case that }X\mathrm{ is true in all states).
```

Note the following points:-
(1) Only actions which are sufficiently "primitive" can be formalised using the above preaicates.
(2) In particular, any variable in an acrion's preconditions must appear as a parameter of that action. This is so that each action has a unique preconditions.
(3) The intended interpretation of 'del' is rather broader than that of STRIPS' delete-1ists. Any fact may be regarded as deleted by an action so long as it is not positively preserved by that action. For efficiency, it is desirable to make 'del' assertions as powerful as possible. For example, we might assert 'del(on(robot, Z), climboff( $B$ ))' if we knew that no fact of the form 'on(robot, $Z$ )' could remain true after the robut has climbed off a box B. It would not be necessary to instantiate $Z$ to $B$, the particular box climbed off.
(4) Similarly, although it is not necessary to specify that certain conjunctions of facts are impossible, this can yield a huge improvement in performance for certain domains.
(5) Finally, notice that a fact cannot be both added and preserved by an action, so if a fact already hoids before $2 n$ action which adds the same fact, the previcus instance should be regarded as deleted. This is important to avoid redundancy - see Section 7.

A particular problem is posed to WARPLAN by a procedure call of the following form:
-plans ( $C, T$ ) : output any plans which achieve the conjunction of facts $C$ from initial state $T$.

## 4. A 5 blocks problem and ifs solution

As an illustration of the preading points, we will give the database for a blocks world, and chen outline how WARPLAN would solve the problem of stacking up 5 blocks. In ctraining the soiution, WARPLAN also solves the 3 blocks problem of Austin Tate。
4.1 The Database

+ add (on(U,W), move(U,V,Wi).
+ add ( clear (V), move $(U, V, W)$ ).
$+\operatorname{del}($ on $(U, Z), \quad \operatorname{move}(U, V, W))$.
$+\operatorname{del}$ ( clear(W), move(U;V,W)).

$+\operatorname{can}(\operatorname{move}(U, V, W) \quad, \quad c l e a r(W) \ell$ on $(U, V) \& U \neq W 8$ clear (U).).
+ imposs (on $(X, Y) \&=1$ eax $(Y)$ ) :
+ imposs (on $(X, Y) \& o n(X, Z) \& Y \neq Z)$.
+ imposs (on $(X, X)$ ).
+ given (start, on(a,floor)).
+ given (start, on(b,floor)).
+ given (start, on( $(, a)$ ).
+ given (start, on(d,flcor)).
+ given (start, on( $\epsilon, \dot{d}$ ) ).
+ given (start, clear(b)).
+ given (start, clear(c)).
+ given (start, clear(e)).



### 4.2 The Problem

- plans (on(a,b) \& on $(b, c) \& o n(c, d) \& o n(d, e), s t a r t)$.

The computation performed by WARPLAN in obtaining a solution can be interpreted as generating a sequence of approximations to the final solution. The first approximation is the initial state:-
(0) Start

We now attempt to solve the first goal 'on(a,b)'. It is not true in 'start', so we attempt to achieve it using the only operator available 'move( $\mathrm{a}, \mathrm{V}, \mathrm{b}$ )', abbrevỉated to 'm(a, $\mathrm{V}, \mathrm{b}$ )':-
(1) start ; $m(a, V, b)$
(Changes to the plan are underifined.) We must now make sure that the preconditions of $\quad \mathrm{m}(\mathrm{a}, \mathrm{V}, \mathrm{b})$ ' are satisfied, inserting extra actions if necessary. The first precondicion, 'clear(b)', is true in 'start', and the second, 'on $(a, V)$ ', is true if 'V := floor', giving the second approximation:-
(2) start ; $m(a, \underline{\text { floor }}, b)$
' $\mathrm{a} \neq \mathrm{b}^{\prime}$ is true (formai inequality of PC terms), so we have one remaining precondition to satisiy : 'clear(a)'. This is not already true in 'start', so we attempt to achieve it using the only available operator:(3) start ; $\mathrm{m}(\mathrm{U}, \mathrm{a}, \mathrm{W})$; mía,floor, D$)$

All the preconditions ars satisfied immediately if we choose suitable instantiations:-

We have now achieved the first miin goal and so far the steps have been identical to STRIPS. The sesond goai 'on(b,c)' is not true in the state produced by the plan so far, so we attempt to achieve it using the only available cperatoz 'm(b, $\left.V_{i},\right)^{\prime}$ '. As STRIPS, we first try to introduce the new action at the end of the suerent plan. However, we notice that a precondition of 'm( $b, V_{1}, c$ ') is 'clear (b)' and this is inconsistent with the already achicved gial 'on(a,b)'. Hence the action cannot be introduced at that point. We next try traving back through the current plan trying to find a suitable point to irisert the astion, taking care that the goal we are achieving, 'on(b, e)', is not deieted by any already generated actions to the right. We find that a possible point to insert is immediately before the last action:-
(5) start ; m(c,a,floor) ; m(b, $\mathrm{V}_{\underline{\prime}, \mathrm{c})}$; m(a,floor,b)

All the precondicions of 'm(b, $V_{:}, c$ )' are satisfied at the point of insertion if ' $V_{1}$ := fiocr' :-
(6) start ; m(c,a,fioor) ; m(b, fiooc,$c) ; m(a, f 100 r, b)$

We have now solvad the first two main goals, so we have a solution to the 3 Bloeks Problem. The prinzipal steps remaining to the final solution of the 5 Blocks Problem are listed below:-



$$
m(b, f 10 c r, c) ; \text { mía,floor, b) }
$$

(9) start ; m(c,a,flcor) ; m(e,d,ficor) ; m(d,fioor,e) ; $m(c, f i o u r, d) ; m(b, \pm 100 r, c) ; m(a, f 1 c 0 r, b)$

The present implementation reaches this solution in a total of 52 seconds CPU time. The search strategy used at present is purely depthfirst, with conventional back tracking. The steps described above, assume that the order of main goals, operator preconditions, and 'tcan' assertions are as presented in Section 4.1. Notice that the above solution is obtained essentially without backtracking, and is not quite sptimal. Further soilitions, including the optimal one, could be obtained if different choices were made at the choice points. Notice that the original goals are achieved in the plan in the reverse order to that in which they are stated, but they are solved by the plan generator in the original order.

## 5. IMPLEMENTATION OF THE SYSTEM

Note: The reader is recommenaf to examine the examples in Appendix II before proceeding with this Section. Refer to Appendix I for a complete listing of the program and to Appendix III for details of PROLOG. It may weil be easier to understand the program listing that this expianation:

The central predicate is ' $p$ lan $\left(C, P, T, T_{1}\right.$ )', the procedure entry point to the main recursive loop.

It has four arguments:-
C is a conjunction of goals to be solved;
$T$ is an (already generated) partial plan;
$P$ is a conjunction of goals already solved by $T$ which must be protected;
$T_{1}$ is a new plan, which contains $T$ as a subnlan and preserves the already solved goals $P$, and which also solves the new goals $C$.

As interpreted by PROLOG, $C, P, T$ behave as input variables and $T_{1}$ as an output variable. The clauses defining 'plan' are:-
$+\operatorname{plan}\left(X R_{1} C, P, T, T_{2}\right)-1-\operatorname{solve}\left(X, P, T, P_{1}, T_{i}\right)-\operatorname{plan}\left(C, P_{1}, T_{1}, T_{2}\right)$, $+p l a n\left(X, P, T, T_{1}\right)-\operatorname{solve}\left(X, P, T, P_{1}, T_{1}\right)$,

Essentially this states that a 'plan' can be produced by 'solve'-ing each goal in the order given. The effect of the '-./' is to tell PROLOG not to consider the second clause if it has successfully "marched" the first literal of the first clause. In this case it could be omitced without affecting the semantics of the program; it is needed to prevent the substantialinefticinacies of trying to 'solve' a conjunction of goals, which is in fact impossible.
'solve $\left(X, P, T, P_{1}, T_{1}\right)$ ' is true if:-
$X$ is an atomic goal;
$T$ is a partial plan;
$P$ is a conjunction of goals achieved by $T$;
$T_{1}$ is a plan, containing $T$ as a subplan, which solves $P_{1}$;
$P_{1}$ is a conjunction comprising $P$ and $X$ where $X$ is not repeated.
$X, P, T$ will be input variables and $P_{1}, T_{1}$ output.
There are three ways in which a goal may be 'solve'-d:-

+ solve ( $X, P, T, P, T$ ) - always ( $X$ ) .
$+\operatorname{solve}(X, P, T, P, T)-\operatorname{holds}(X, T)-\operatorname{and}\left(X, P, P_{1}\right)$.
$+\operatorname{solve}\left(X, P, T, X \& P, T_{1}\right)-\operatorname{add}(X, U)-\operatorname{achieve}\left(X, U, P, T, T_{1}\right)$,
It may be 'always' true in the world. It may be that it already 'holds' in the state produced by the current partial plan. Finally we may look in the database for an action $U$ which 'add'-s the goal $X$ and then 'solve' $X$ by 'achieve'-ing U .

There are two methods to 'achieve' an action, which we will call extension and insertion. If we were to omit the clause for insertion, we would get a system almost identical to STRIPS without the ability to permute goals. The clause for extension is:-
$+\operatorname{achieve}\left(X, U, P, T, T_{1} ; U\right)$

- preserves ( $U, P$ )
- can (U, C)
- consistent ( $C, P$ )
- plan $\left(C, P, T, T_{1}\right)$
- preserves ( $U, P$ ),

We first check that the action $U$ preserves (ie. does not delete) the protected facts $P$. Then we lookup the preconditions $C$ in the database and check that $C$ is consistent with the protected facts. All being well, we call. 'plan' recursively to modify the current plan $T$ to a new plan $T_{1}$ which produces a state in which $C$ is attained as well as $P$. $U$ can then be applied in $T_{1}$, corresponding to the plan resulting from this call of 'achieve'. Finally, we repeat the check that $U$ preserves $P$. The reason for this is that $U$ and $P$ may not have been instantiated to ground terms at the time of the original check.

Lacking the ability to coroutine in PROLOG at present, we have to be satisfied with an incomplete firct check followed by a second check "to make sure". Even so there is still a slight flaw in the program, as $U$ and $P$ may still not be fully instantiated by the time of the second check.

The clause for the second method of 'achieve'-irg an action, insertion, is:-

```
+ achieve(X,U,P,T;V,T1;V)
    - preserved (X,V)
    - retrace( }P,V,\mp@subsup{P}{1}{}
    - achieve(X,U, P1,T,T T )
    - preserved (X,V),
```

If the last action $V$ in the current partial plan joesn't delete the current goal $X$, we can try to insert the action $U$ somewhere before $V$, provided we 'retrace' the set of protected facts to the point before $V$. $P$ is 'retrace'-d to $P_{1}$ before $V$ if

```
P
```

As mentioned previously, in WARPLAN, plans and states of the world are virtually synonymous. Everything that 'holds' in a state of the world can be determined from the plan which produces that state of the world. The system chains backwards through the sequence of actions, so long as none of these actions deletes the sought-for fact, until the fact is found in the 'add'-set of an action or was 'given' in the initial state:-
$+\operatorname{holds}(X, T ; V)-\operatorname{add}(X, V)$,

+ holds $(X, T ; V)-1$
- preserved ( $\mathrm{K}, \mathrm{V}$ )
- holds $(X, T)$
- preserved $(X, V)$.
$+\operatorname{holds}(X, T)-\operatorname{given}(T, X)$.
This method avoids the overhead of generating a net set of facts for each state of the world considered, as do STRIPS, PLANNER \{6\}, etc. albeit in a structure-shared form. However, to balance against this, there is more computation involved in accessing a fact (see Section 6 for possible improvements).

To prove that a fact $X$ is preserved by an action $V$, WARPLAN essentially tries to satisfy itself that it can't prove that $V$ deletes $X$ :+ preserved $(X, V)$ - mkground $(X \& V, 0, N)-\operatorname{del}(X, V)-/-f a i l$. + preserved ( $X, V$ ).
'mkground' substitutes "arbitrary constants" for any variables in the terms currently bound to $X$ and $V$. If we can now prove that $X$ is 'del'-eted by $V$, we call '/' to prevent any further choices being taken for 'preserved' and then call 'fail'. This is an arbitrary predicate which can't be proved true, since we supply no definition for it. The net effect is that the original attempt to prove 'preserved $(X, V)$ ' fails, and subsequent backtracking of course "undoes" the effects of 'mkground'. In the other case we can't prove that $K$ is 'del'-eted by $V$ and the second clause allows the procf of 'preserved $(X, V)$ ' to succeed. Unlike previous uses of '/', this use actually changes the meaning of the two clauses. As a "hack", the technique is rather powerfuJ. and not without a certain appeal.

The remainder of the WARPLAN axioms define some fairly straightforward auxiliary procedures.
6. DEFICIENCIES OF THE SYSTEM

Some deficiencies of the system are listed below. They range from relatively minor details to problems which suggest that a totally new approach is needed.
(1)

The 'holds' axioms could be made more efficient by using knowledge of action preconditions to avoid always chaining back to the point at which a fact was added, as is done for 'inroom(robot, room(1))' in the fourth STRIPS problem (Appendix II, 1.3) for example. If a fact occurs as a precondition of an action in a plan, we know that the fact must hold immediately prior to that action in the plan. So it is only necessary to chain back as far as the last time the fact was "used". Thus every time a fact were "used" it would become more "accessible" for further use. However it is difficult to do this without introducing redundancy.

There is a similar possible efficiency improvement associated with the consistency checks. When an inconsistency has been found, the system should immediately retrace as far as is necessary to
remove one of the already-achieved goals which "cause" the inconsistency.
of clauses with the same leftmost predicate, and not to clauses within such a group. Thus WARPLAN is continually chaining through its entire list of 'add' clauses, for example, to find a suitable action to add a certain fact. More direct access could be achieved with the current PROLOG at the expense of some loss of clarity in the WARPLAN axjoms. This should yield a substantial improvement in speed.

The system needs a more intelligent search strategy. Most of the problems which it has solved involved little or no backtracking. Backtracking frequently results in crazy alternatives being tried next. This arises particularly because there is nothing to prevent WARPLAN from constructing a plan to achieve a fact which is already true in the current state. (This facility is needed, in some cases at least, for completeness.) .

An automatic check for loops could alleviate this and other problems, but would probably slow the system down substantially.

At the moment, goals, action preconditions and 'add' clauses can be pre-ordered by hand to give the best results. It would be nice if the system could do the analysis necessary for this (cf. the way LAWALY determines its hierarchies). It would of course be better if the orderings were determined dynamically.

Actions which add several facts frequently need "augmented" preconditions for some of them. (Two versions of 'take' were used in the Keys and Boxes Problem (Appendix II.2) to bypass this problem.) It should not be too difficult to provide this facility.

There are a number of ways in which PROLOG might be enhanced to WARPLAN's benefit. The inability to put "restrictions" on variables results in some flaws in the program. This is a special case of the need for co-routining - more flexible choice of which litexal to cancel next (ie. which goal to solve next).
(9)
with the goal:-
inroom(rcbot,rom 2) \& nextto(robot,box3)
for which the optimal zuiation might be:-
start ;
gote(bex 3)
shunt thru(box 3,dose, room 1,room 2) ;
goto (box 3, room 2)
where 'shuncthru' does not leave the robot 'nextto' the shunted object. WARPLAN ras to firet produce a complete solution to one of the two top-ievei goals, and, for it to subsequentiy find the optimal soiution, it is recessazy that rhis partiai silution be a subpian of the optimal pian . Sippose the top ísvel goals are ordered as above. Then the initial subpian needed for 'inrocm(rubst, room 2)' is:-
start; goto(ocx 3); shuntthru(bcx 3, azor, room 1, room 2)
a rather unlikely solution' (We are assuming there is a "gotion" operator, and of course there are sevezal boxes.) If, on the other hand, 'nextto(rcbot, bcx 3)' is ordered tirst, then the partial plan neaded is the complete plan itself! The problem is that WARPLAN is generating too much detain in its solution to one subgol before going on to eensider a depenjent subgoal.

Like STRIPS, WARPLAN gentaces a plan by a mixture of backward (from the goal) and forward (from the ithtial state) analysis. For many problems, particuiacly "dititicult" ones suih as in (9) above or
block stacking or imposibibie tasks, it appears that a completcly backward analysis solves the problem better. One starts with the given conjunstion of goals and applies operators "in reverse" to generate a new conjunction of goals. Each new conjunction of goals is checked for consistency, and possibly for subsumption by other conjunctiuns of geais which have been generated. A solution is found when the conjunction of guals is satisfied in the initial state. The problem with this technique is that mere and more variables get introduced into suasessive gals and these need to be restricted in complex ways; the initial state only gets "used" in the final step (although it would peasumady diract the search strategy). The advantag of the SIRIPS-iike analysis is that variables get quickly instantiated making the system much more amenabie to implementation in the present PROLOG.
(11) WARPLAN is unable to generate conditional plans; nor is it therefore able to generate iterarive plans.

Plans generated by WARPLAN are totaliy ordered, often arbitrarily. Besides unnscessarily zescricting the freedom of the plan executer, this also means that there is potential redundancy in the search space,
7. COMPLETENESS AND 1KREDUNDANCY
7.1 Preliminary Definiticus

For definitions of added, preservad, preconditions, see Section 3.
A plan of langch $N$ compriミes an initial stata $T_{0}$ and a sequence of actions $A_{1}, A_{2}, \ldots, A_{N}$. It will be written ' $T_{0} ; A_{1} ; A_{2} ; \ldots ; A_{N}$ '. We require that the plan be executabie, ie, the state resulting from ' $T_{0}$;...j $A_{I-1}$ ' satisfies the preconditions of $A_{I}$, for $I$ from 1 to $N$, A problem is a pair $\quad C, T_{C}$, comprising a conjurction $C$ of facts which are main gosis and an initial state $T_{0}$. A plan $T$ solves the problem : $C, T_{0}$ " if $T_{0}$ is the initial state of $T$ and $C$ holds in the state resulting from $T$,

A plan is optimal fur a problen if there is no plan of lower length which soives the probiem.

A pian generator is smpieṫ it it will oventudily benerate an optimal plar for any probiäm，

A pian is minimil（for a probiem）if єoery action io needed．An action in a plan is ne三ácu it it adds a fact which is a main goal or a preccadicior of a needed aztion．

A plan $T$ is a subpan or d pian $T_{1}$ if
（i）$T$ and $T_{2}$ hate ihe sume ititial state
and（iij）the acthons cf $T$ are a subset of the actions of $T_{i}$ 。
$T$ is a proper subplen of $T$ ，is the actions of $T$ are a proper subset of the actions of $\mathrm{T}_{2}$ ．

## 7．2 Coroilaries of the Defirijtlons

Any plan which sidues a problem has a unique minimal subplan which solves the problem．（It is unique bacause we do not allow a fact to be both added and preserved by an actien．）

An optimil pian musc de minimal．
A plan generatcr which ©an generate ail minimel plans is complete．

## 7．3 Examp1E

Given the problem：－

a far from optimal plan which does in iact solve the problem is：－ start ；movele，d，fixur）；move（d，flour，b）；move（e，a，d）；muve（a，floor，of

The correspcnding minimal subpian is：－
stare ；move（e，d，fioor）；mave（c，x，d）；ače（a，fioor，c）
［＇move（a，ficor，c）＇is nèded since ir ähieves＇on（a，c）＇，ध
＇move（c，a，d）＇is meaded sime ic aこhieves＇elear（c）＇which is a precendition of＇move（a，tiocr，c）＇which is needed，
．＇＇move（e，d，fiours＇is netded since it achieves＇Aiear（d）＇which is a precondicicn of＇mive $(x, \bar{a}, a)^{\circ}$ 。
The cniy action net needed is＂maie（d，ticor，bj＇？］

### 7.4 Outline of a Complatenes. Proot

The compieteness of WAKPLAN ficilows from the fact that 'plan ( $C$, true, $T_{0}, T$ )' is a valid deduction from the WARPLAN axicms and pioblem database if and only if $T$ is a minimal plan which solves the problem <C, $T_{0}>$. To get a complete implementation of WARPLAN, one would need to
(1) provide a propex implamentation of formal inequality ("restrictions" on variables) in PROLOG and modify the clauses for 'preserved' efc. to take advantage of this;
(2) make the PRCLOG saarch sirategy complete. For instance, every time a chuice point is encountered (more than one input clause maiches the current literai) set up the different choices as "parallel (independent) processes". In outlining the proof, we shall only discuss the part played by the piemary clause: , those 1abeiled P1, P2, S1, S2, S3, A1, A2, H1, H2, H3 in Appendix I. We assume that the secondary clauses (the remaining WARPLAN ciauses and the problem database) are a complete and correct iormulacion of their intended interpretations.

We wish to show thac, for any problem $\left\langle C, T_{0}\right\rangle$, given a minimal plan $T$, there exists a derivation of 'plan ( $C$, true, $T_{0}, T$ )' from the WARPLAN axioms and problem database.

A derivation of a fact $X$ from a set of Horn clauses $S$ is a tree of instances of clauses from $S$, such that
(1) the top (or root) clause instance has $+X$ as its positive 1iteral;
(2) for each negative iiteral - L occurring in a clause instance, there is exactiy one subtree at that node which is a derivation of $L$.

A clause is a Hozn clause if it has at most one positive literal.
The proof proceeds by induction on the length of the minimal plan $T$. The proposition for procf by indaction is:-

For any $C=G_{2} \& G_{2} \& \ldots \& G_{M}, T_{0}$, and minimal plan $T$ of length $N$, there exist derivations of 'solve $\left(G_{I}, P_{I-1}, T_{I-1}, P_{I}, T_{1}\right)$ 'for Ifrom 1 to M
where

$$
\begin{aligned}
& P_{2}=\text { řrlé, }
\end{aligned}
$$

$$
\begin{aligned}
& T_{I} \text { ie theminamal zubpian of } T \text { which sulves } P_{i} \text { 。 }
\end{aligned}
$$

 trivialiy．）

## Case $N=0$ ．

The propisicion holds ctiviaily asing iristances of aizuses S1 and S2．
We must now asscme the prepositurn heds ios $N$ End prove if for $N+1$ ． We shail meitiy destabe the jonctrution needed tor this step．A rigorous proof woul involve theckirg at the details．

Let $T=\left(T^{\prime} ; U\right)$ ．

Let $G_{I}$ be the first sach giel．
Let $K_{1} \&_{1, \ldots}, K_{j}$ be zint gicis of $C$ which appiar after $G_{I}$ in the conjunction and which aze nct anhieved by $U$ ．
Let $H_{2} \& H_{2} \& \ldots \& H_{I}$ bé the paezonditions af $U$ ．
Let $C^{\prime}=G_{1} \&, \ldots \& G_{I-i}$ \＆$H_{2} \&, \ldots \& H_{L} \& K_{:} \&:, \&_{J}$ Then cleariy $T^{\prime}$ is a minimai pian for rine probiem $\left\langle C^{\prime}, T_{0}\right\rangle$ 。 By the ind：ative nyporiasiss there exises a derivatizn of ＇plan（ $C^{\prime}$ ，titues $\left.T_{0}, T^{\prime}\right)^{\prime}$ ．We snail shisw how to use this derivacion to anstrict a desivation of＇pian（ $C$ ，true，$T_{0}, T$ ）＇． The eonstruttion is Alustacted below：－


A derivativit is a the of instances of WARPLAN Clauses.
We ignore instances of seavacty clauses.
It is clear that televise P, and P2 serve only co Jink together a number of derivations ot "suse' Iftesalso we indicate mucin an occurrence at a set of $P_{i}$ ilsuscs with Ene $P 2$ clause by 'P1/P2'. A subarea is imaicased by a tifiangle。 Thu sEe in the diagram above represent derivations of 'solve $(X, \ldots$ )' where $X$ is the label inside the triangle. Must of the subtrees in the "output" derivarioti are $\quad$ apia finn the corresponding oubtaee in the "input" derivation. It reanine to show how the subareas for $G_{I+1}$ io $G_{j}$ are constructed:-
[if $G_{x}$ is a gusil achicwed by $U$ then

else $G_{x}$ has been given the name $K_{y}$ for some $y$;

is of theformi

 is of the form
 is of the form

]
]

That completes the outline of the proof.

### 7.5 Irredindancy

A plan generator is iriedunami if, in generating plans, it never generates the same plan more than once.

A set: of (Horn) aliases $S$ is ireejundant, if for any fact $X$, there exists at must one derivation of $X$ from $S$.
 to the irfedundancy at a ser uf dizus三s vompasing the Wheplan axioms and an irredundant probem databses．In last wakplan í irsedundant provided that

（2）＇adf $(X, U)$＇and＇p．estoved $(X, U)^{\prime}$ are mainaily exaivsive；


Unce again we shai inty onsidec the psimary clojses，assuming the secondary＝iauses are irreandaant．We have to show that it is
 where $C, T_{0}$ l are gronat termi．

Fox most of the pansay preaicates，it is cleax by inspection that， for ground instantistiens of fre first and Last arguments，there is orijy cne clause whith ian be usea tu dexive thst literal；giverthe assumptions listed abure。

The unly cese ar whizh there is dirficulyy is for the clauses $S^{2}$ and S3．Suppase they var buth oe ioca to àrive the same jiteral ＇solve $\left(X, P, T, P_{2}, T_{i}\right)$＇，where $X, P, T, P_{1}, T_{2}$ dre ground teme．From S2，$T=T_{1}$ 。 But ansiterang $S 3$ ，it is iexin that a dexivation of the＇achieve＇literal is oniy pussibie it the lengrn if $\mathrm{T}_{1}$ is at ledst one greater then $T$－a contáedineion．

Therefure che ižedundercy hás been demonstrated．

## 7．6 Anseher Exjmp？e

To furcher eluzidame the ampleteness of WarPLAN，the diagram belaw indiades how whfian would ariva at the optimai solucsen to the biceks Problem（the eoturion desuibed in Section 4 is the one which would be found first by a depuh－tirstevach stionegy），

The diagram represants the situation cyleulus prosi of the optioni plan which is impilicty geateated by wakiad．A biack avt represents an addfyeconditicns dxiom wheress a white dor cepresents a frame dxiom， The numofs labiaileg each ast indieate the dader in whith the proof is buils up．In＝prof it rhe prounditions of an operator is only
 antion is indiadea by ar upwacd atcu：


- Preoonditions (in order
given)


In crder to find this pioos, wakfan has ficlect to 'cleariaj' by a

 although the optimai soiution is in the search space, it is discovered by a rather unnadurs? process.
8. CONCLUSIONS
8.1 Predicat Calcaies as Pragranming Language

WARPLAN is a program impiemented in Fredicate Caicalus. I believe the use of this langizge made the frogram shoter, dearer and easier to implement, with littie (if any) sacrifice of efficiency. I doubt whether a shorter program cuala be psoduced by rewriting the aigorithm in another programming langajge。

More generaily, I suppart Kcwaiski's \{8\},iio\} contention that PG is a superior high-level prcgamming language suitabie for wide use, particularly in AR. Expezience winh PROLOG asgeste that any program that can be formatata in LISP can be formulaíad at least as easily in PRCiOG. Frequently the naturainess of $F C$ can suggest a mare elegant formulation.

So far this is only assuring an interpecter such as PROLOG with an unscphiscicatica (bat fist) procif prozedure, which gives PC programs a rather anventions1 cyncrei struature. Inference bystems oush as Kowalski's Cunnection Graphs \{9\} sugbest how, in principle, control structures might be iiberaides to geat perenticil advertage, but much remains to be done ro make chis a practical yeality.

Wherein iifs the adrantage of FC over other ianguages? One might attribute it to the feacures which PC doesn't provias. High-1evel languages have progeessed by successively uransferring responsibilities from the user tis the softwaze engineer. The user is reifieved of certain machine-oriented tisks which wjuid mike his prcgram elumsy to formulate and difficale to detog, leaving him free to concentrate on the problem-oriented aspects. One of the first sioh tajks to come under software control was the mapping if data into aetual machine addresses. lauer advances made the early machine-oriented censept if a goto redundant (though still widely used to the detriment of clear prcgrammirg).

I believe a similas :itation exisis with zega:d to assignment (the ability to "re-use" a variabje to dente more than one vaiue during the course of a computation). Experience with AJGOL 68 sonvinced me that true assignment is sarely needed, and its avoidanceresults in programs which are easier to understand. There is of course no assignment in PC, and it's ail the better te: it:

Among the nice features of PC which might be mentionea aite:-
(1) one langage tor paugram and data;
(2) the usadi tree data-structures, but with a beautituly natatal woy to manipulate them via the generality of unification;
(3) no explicit distinciion between input and output variables, so that a sing ie predicate may furction as several differenc procedures.

As an illustration of the elegance of PC programing, consider the definition of 'intersect' from Appendix I :-

+ interserc $\left(S_{1}, S_{2}\right)-$ Eient $\left(X, S_{1}\right)-\in 1 e m\left(X, S_{2}\right)$
This simply makes precise the natural language stacement that "two sets intersect if they have an element in common". The detiation is conceived before sorisideration is given to its procedural asperts iso how it will be used by the PRCiOG interpzeter. In fact, given fail instont acions for $S_{1}$ and $S_{z}$, PROLOG will proceed to generate nom-deterministicaily the elemente of $S_{1}$ and then check to see whether they are elements of $S_{\text {: }}$ (ie, 'elem' is functioning as two discinct provedares). This example demonstazas the closeness of PC te natural ionguage. The much-mitigned claussl form seems quite natural when $P C$ is $g$ i\%en a procedural ineerpretation.

An interesting aspect of PC is the way crie set cif lizuses really represents an equivalence dias of different algorithms. For exampie, in . Appendix $I$, if we interchinge the two clauses for 'achieve", PROICG gives a different "version" of Warplan in which actions are (first) inserted as far left as possible in the current plan, rather than tirst trying to tag the new action on to the end of a plan. Similarly, changing the oroer of literals in a clacse can projuce ditferent but ceasonabie algorithms.

A factor hindering the progress of AI seems to be that Ail programs are not produced in a fuim suitable for easy comprehension by humans. The essential ideas urderlying the pregram become obscured when it is "coded". Consequently the programs chemselves are rarely publi:ned, but are onty described, leading ine itsbiy to vagueness and ambiguity. A case in point is the literature on STRIPS, which by comparison wich wist fapers is a model of clarity, and yet the number of hours I know I and ochers heve spent
in arguing about "what STRIPs tealiy does" ...
Programs need to be pubished. For this cne tequires a language with the simplisity and univessility of PC. The ain of programing should be towards an ideai of beauty and ciarity. A progian should not have to be "explained" by flowhates or interspersed comments.
8.2 Lugic in Logic

The WARPiAN progrem leas fC terms to repeesent conjuncions of facts (or gozls). Thas the ubjects of diszourse inciade sentences of logic. In addition, there is a one-to-one corrsspenaence between pians and the proofs that these plans anheve the desired goals*. (The "proof" of a plan is, as in Section 7.6, a tree conctructed from instances of 'adid' axions, 'can' axions and implicit mane axioms derived from the 'dei' axiomso) Thus the terms representing plans can equally weil be said to represent proots of these pians; the cbjects of discourse include proofs of sentences of logiz.

Accorditgly, WARPLAN miy be described as a theoram-prever implemented in a theorem-prover. (if. fur example, an ALGOL compiler writien in ALGOL, I believe this techniqie may have considecable potential and answers to some extent the criticism that Higher Order Logic is needed to express certan problems.
8.3 The Utility of Inconsistency Tests in Platning

WARPLAN can make effective use of negative statements about the worid Typically thase statemerts of impossibility refiect the "physics" of the world whereas the actions represen: "engineering" in accordance with this physics. In partioulaz the "delere list" of an action ought to be computable from a knowledge of the physics of the worid, rather than being spelled out explicitly. (cf the 'dei' facts for the Machine Code
Generation problem in Appendix II.3.2,)
The positiva and negative aspects of problem statement and problem solving merit further investigation. The system Disprover of Siklossy and Roach \{15\} tries ic show that a goai is an eagineering impossibility whereas WARPLAN can only look for physical impossibilicies. (A man on the koon was an erigincering impossibility lase century but not a physical

[^1]impossibility, like perpetuai mution; it only needed a few more operators to be added to ihe world:'

### 8.4 The Frame Problem

The frame prublem \{12\} \{5\} hiss frequently been cited as an argument against the use of "uniform proof procedures". Since it is hard to imagine a proof procedure more "uniform" than PROLOG, it is interesting to consider whether or not the frame problem arises when the WARPLAN clauses are intexpreted.

As Kowalski has poiriced out $\{10\}$, there are two aspects to the frame problem as it spplies to the traitional situation calculus formulation, The first is the inconvenieace cf having to explicitly state ail the frame axioms. The second is the problem of inefficiency arising from the frame axicms.

The approach to the first paublem in WARPLAN is much the same as in STRIPS. One specifiea ungy which facts are deleted by an action and assumes that all other facts ace preserved. Moreover, in WARPLAN, the specification doesn't have to be explicit - delate information may be represented procedurally. Notice however that frame axioms are still used implicitly in WARPLAN's deductions. For example, the second 'holds' clause in Appendix $I$ is cleariy a universal frame axicm which can become instantiared to different fiame axioms for varicus actions.

The efficiency problem is tackied by:-
(1) only using the frame axioms "top-down", and
(2) the ability to (impliaitiy) insert frame axioms into the proof of the current partial plan.

The "top-down" use of a frame axiom centrasts with a "buttom-up" use in which it is used to generite facts ábuat a new state. In WARPLAN a fact about a state is only deduced when meeded from the history of the state. Tc see how WARPLAN inserts frama axicms into a proof, consider say SL resolution $\{7\}$ cperating with the goai state clause of a situation calculus problem as top ciause. In carmeliing, the selected literai, it has to thoose between an "add" axiom or a frame axiom, and the choise is irrevocable (without backtracking). In the analagous situation in WARPLAN, oniy an "add" axiom cculd be chosen. Later, and without backtracking, any number
of frame axioms could be inserted between the negative goal literal and the positive literal of the "dda" axiom.

There are still, however, difficulties which might be ascribed to the frame problem, in particulat the redundancies and inefficiencies in solving a conjunction of goais relating to "independent sub-worids". I think it is important to note that the frame problem is not peculiar to PC formulations. There are analogues of frame axioms in all systems which maintain a record of more than one state or context with information structure-shared between them.

## A P P EN D I X I. LISTING OF THE PROGRAM.

NOTE: the labels "P1:" etc. are referenced in Section 7 and are not part of the program.

+ operator ('\&', right to left, l).
+ operator (';', left to right, 3).
$P_{1:}+\operatorname{plan}\left(X \& C, P, T, T_{2}\right)-/-\operatorname{solve}\left(X, P, T, P_{1}, T_{1}\right)-p \tan \left(C, P 1, T_{1}, T_{2}\right)$,
$P 2:+p l a n\left(X, P, T, T_{1}\right)-\operatorname{solve}\left(X, P, T, P_{1}, T_{1}\right)$.
$S_{1}:+\operatorname{solve}(X, P, T, P, T)-\operatorname{always}(X)$,
S2: $+\operatorname{solve}\left(X, P, T, P_{1}, T\right)-\operatorname{holds}(X, T)-$ and $\left(X, P, P_{1}\right)$,
S3: $+\operatorname{solve}\left(X, P, T, X \& P, T_{1}\right)-\operatorname{add}(X, U)-\operatorname{achieve}\left(X, U, P, T, T_{1}\right)$,
$A 1:+\operatorname{achieve}\left(X, U, P, T, T_{1} ; U\right)$
- preserves ( $U, P$ )
$-\operatorname{can}(U, C)$
- consistent ( $C, P$ )
- plan ( $C, P, T, T_{1}$ )
- preserves ( $U, P$ ),

A2: + achieve $\left(X, U, P, T ; V, T_{1} ; V\right)$

- preserved $(X, V)$
- retrace $\left(P, V, P_{1}\right)$
- achieve $\left(X, U, P_{1}, T, T_{1}\right)$
- preserved ( $X, V$ ).
$H 1:+\operatorname{holds}(X, T ; V)$-add $(X, V)$.
$H 2:+\operatorname{holds}(X, T ; V)-/$
- preserved $(X, V)$
- holds (X,T)
- preserved $(X, V)$.
$H 3:+\operatorname{holds}(X, T)-\operatorname{given}(T, X)$.
$+\operatorname{preserved}(X, V)$-mkground $(X \& V, 0, N)-\operatorname{del}(X, V)-1-$ fail.
+ preserved $(X, V)$.
$+\operatorname{preserves}(U, X \& C)$ - preserved $(X, U)$-preserves $(U, C)$ :
* preserves ( $U$, true).

```
+retrace ( }P,V,\mp@subsup{P}{2}{}
    - can (V,C)
    - retrace 1 ( }P,V,C,\mp@subsup{P}{i}{}
    - append (C, P, , P2).
+retrace 1 ( }X&P,V,C,\mp@subsup{P}{1}{}
    - add ( }Y,V)\mathrm{ - equiv ( }X,Y\mathrm{ )-/-retrace 1 ( }P,V,C,\mp@subsup{P}{1}{\prime})
+ retrace 1 ( }X&P,V,C,P\mp@subsup{P}{1}{}
    - elem (Y,C) - equiv ( }X,Y\mathrm{ ) -/- retracel ( }P,V,C,\mp@subsup{P}{1}{})
+ retrace 1 ( X & P,V,C,X & P ) -retrace 1 ( P,V,C, P 1 ),
+ retrace l (true, V, C, true).
+ consistent ( C,P)
    - mkground (C & P, O,N)
    - imposs (S)
    - unless (unless (intersect (C,S)))
    - imp1ied (S,C &, P).
    -/- fail.
+ consistent (C,P),
+ plans (C,T) - unless (consistent(C, truej) -/- output (impossible)-newíne.
+ plans (C,T)-plan(C,true, T, T ) - output( }\mp@subsup{T}{1}{})\mathrm{ - newline.
+ and ( }X,P,P)-elem (Y,P) - equiv ( X,Y)-/.
+ and ( }X,P,X&P)
+ append ( }X&C,P,X&,\mp@subsup{P}{1}{})-/- append ( C, P, P P )
+ append ( }X,P,X&P)
+ elem (X,Y & C)-elem ( }X,Y\mathrm{ ),
+ elem ( }X,Y&C) -/- elem ( X,C).
+ elem (X,X).
+ intersect (S S, S ) - elem ( }X,\mp@subsup{S}{1}{})=\operatorname{elem}(X,\mp@subsup{S}{2}{})
+ implied ( }\mp@subsup{S}{1}{&&S S , C) -/-implicd ( S S , C)-implied ( }\mp@subsup{S}{2}{},C)\mathrm{ .
+ implied ( }X,C\mathrm{ ) - elem ( }X,C\mathrm{ ).
+ impiied (X,C) - X,
```

```
+ equal ( }X,X\mathrm{ ),
+ notequal ( }X,Y\mathrm{ ) - vules; (equal( }X,Y)\mathrm{ )
    - unless( (qual (X,qqq(N.)))
    - unless (equal (Y,qqq(N,))).
+ equiv ( }X,Y\mathrm{ )-uniese (monequiv( }X,Y)\mathrm{ ).
+ nonequiv ( }X,Y\mathrm{ Y ) -mkgzound (X& Y,O,N)-equal (X,Y)-/-fail.
+ nonequiry ( }X,Y\mathrm{ Y).
+ mkground(qqg(N,);N2,N N -/- puus(N, (N,N2)。
```




```
+ mkgroundiist ( ( ( X,i),N, N, N
    -mkground (X,N, N N )
    -mkgroundicet (L,N2,N_),
```



```
+ un1ess(x)-x-i-fail..
+ un1ess (X).
```


## A P P EN D I X II. TEST PROBLEMS

## . 1 First STRTPS Wor1d

## .1.1 Description

The problem domain and its formalisation are essentially the same as that given in the original $\operatorname{STRIPS}$ paper $\{3\}$. The chief differences are that the delete lists and certain other facts are represented procedurally and various "types" checks are nandled automatically by the unification algorithm. The domain is of interest for comparing the performance of WARPLAN with similar planning systems:-

| System | Implemented In | Machine |
| :--- | :---: | :---: |
| WARPLAN | PROLOG (interpreted | In Fortran) |
| STRIPS | LISP (partially compiled) | PDP 10-67 |
| LAWALY | LISP (interpreted) | CDC 6600 |

The times quoted below are total CPU times. The STRIPS times exclude garbage collection. PROLOG does no garbage collection (although space is reclaimed on backtracking) as the implementation is carefully designed to conserve storage. What weightings, if any, to apply to the different figures seems to be largely a subjective matter.

## .1.2 Database

```
+ add( at (robot,P),
+ add( nextto(robot,X),
+ add( nextto(X,Y),
+ add( nextto(Y,X),
+ add( status(S,on),
+ add( on(robot,B),
+ add ( onfloor;
+ add(inrcom(robot, R2),
```

```
goto I (P,R) ).
```

goto I (P,R) ).
gotc 2(X,R) ).
gotc 2(X,R) ).
pushto (X,Y,R) ).
pushto (X,Y,R) ).
pushto (X,Y,R) ).
pushto (X,Y,R) ).
turnon(S) ).
turnon(S) ).
clinabon(B) ).
clinabon(B) ).
climboff(B) ).
climboff(B) ).
gothru(D,R}, ,\mp@subsup{R}{2}{}) )

```
gothru(D,R}, ,\mp@subsup{R}{2}{}) )
```

```
+ del ( at ( }\textrm{X},\textrm{Z}),U)-moved (X,U)
+ de1 ( nextto(Z,robot),U) -/- del(nextto(robot,Z),U).
+ de1 ( nextto(robot, X),pushto(X,Y,R)) -/- fail.
+ de1 ( nextto(robot,B), climbon(B))-/-fail.
+ de1 ( nextto(robot,B),climboff (B))-/- fail.
+ de1 ( nextto (X,Z),U)-moved (X,U).
+ de1 ( nextto (Z,X),U)-moved ( }X,U)\mathrm{ ,
+ del ( on ( }X,Z),U)-\operatorname{moved}(X,U)
+ del ( onfloor,climbon (B)).
+ del (inroom(robot,Z),gothru ( D, R1, R2).
+ de1 ( status(S,Z),turnon(S)).
+ moved (robot,goto 1 (P,R)).
+ moved (robot,goto 2 (X,R)).
+ moved (robot,pushto (X,Y,R)).
+ moved (X,pushto (X,Y,R)). .
+ moved (robot,climbon (B)).
+ moved (robot,climbcff (B)).
+ moved (robot,gothru(D, R1, R2)).
+ can (gotol (P,R),
    locinroom ( }P,R\mathrm{ ) & inroom(robot,R) & onfloor).
+ can (goto 2 ( X,R),
    inroom (X,R) & inroom(robot,R) & onfloor).
+ can ( pushto(X,Y,R),
    pushable (X) &inroom ( Y,R) & inroom (X,R) &
    nextto(robot,X) & onfloor).
+ can ( turnon(lightswitch (S)),
    on(robot,box(1)) & nextto(box(1),1i.ghtswitch(S))).
+ can ( climbon(box(B)),
    nextto(rcbot,box(B)) 足on floor).
+can (climboff(box(B)),
    on(robct,bcx(B))).
+ can (gothru(D, R1, R ) ,
    connects( }D,\mp@subsup{R}{1}{},\mp@subsup{R}{2}{})\mathrm{ &inroom(robot, }\mp@subsup{R}{1}{\prime}\mathrm{ ) &:
    nextto(robot,D) & onfloor).
```

```
+ always \(\left(\right.\) connects \(\left.\left(D, R_{1}, R_{2}\right)\right)-\) connects \(1\left(D, R_{1}, R_{2}\right)\).
+ always (connects \(\left.\left(D, R_{2}, R_{2}\right)\right)\) - connects \(1\left(D, R_{1}, R_{2}\right)\).
+ always (inroom \((D, R))\)-always (connects \(\left(D, R, R_{1}\right)\) ).
+ always (pushable (box (N))).
+ always (locinroom (point (6), room (4))) .
+ always (inroom (lightswitch (1), room (1))).
+ always (at (lightswitch (1), point (4))).
+ connects 1 (door ( \(N\) ), room ( \(N\) ), room (5)) - range ( \(N, 1,4\) ).
+ rangé ( \(M, M, N\) ),
\(+\operatorname{range}(M, L, N)-\operatorname{notequal}(L, N)-\operatorname{plus}\left(L, 1, L_{1}\right)-\operatorname{range}\left(M, L_{1}, N\right)\),
\(+\operatorname{given}(\operatorname{strips} 1, \quad\) at (box \((N)\), point (N))) -range (N, 1,3\()\),
+ given (strips 1, at (robot, point (5) )) 。
+ given (strips 1, inroom (box ( \(N\) ), room (1)) )-range ( \(N, 1,3\) ),
+ given (strips 1 , inroom (robot,room (1))).
+ given (strips 1, onfloor).
+ given (strips 1, status (lightswitch (1), off)).
```

| strips 1: | room (1) | room (2) | room (3) | $\underbrace{\text { room (4) }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | ```dloor:(1) room (5)``` | door (2) | door (3) | door (4) |

```
.1.3 Problems
    Goa1 (s)
(1) status(1ightswitch(1),on)
(2) nextto(box(1), box (2)) & nextto(box(2), box (3))
(3) at(robot,point(6))
Plan CUP time in secs.
length WARPLAN STRTPS IAWALY
\begin{tabular}{cccc}
4 & 9 & \(65 *\) & 1.6 \\
4 & 21 & 122 & 4.1 \\
5 & 9 & 125 & 2.6
\end{tabular}
(4) nextto(box(2), box(3)) \& nextto (box(3), door(1)) \&
        status(lightswitch(1),on) &
        nextto (box(1),box(2))& inroom(robot, room(2)) 15++ 95+ - 10.3**
    * :non-optimal solution
    ** :to produce a similar 15-step plan.
    + :time to produce all but the last step, at which point, apparently,
        an error in PROLOG eaused the system to crash.
    ++ :solution produced by WARPLAN is:-
                strips 1;
                    goto 2 (box(3),room(1)) ;
                    pushto (box(3), door(1),room(1));
                goto 2 (box(2),room(1) ) ;
                pushto (box(2),box(3),room(1)) ;
                goto 2 (box(1),room(1));
                pushto(box(1),lightswitch(1),room(1)) ;
                climbon(box(1) ) ;
                turnon(lightswitch(1));
                climboff(box(1));
                *goto 2 (box(1),room(1)) ;
                pushto(box(1),box(2),room(1));
                goto 2 (door(1),room(1)) ;
                gothru(door(1),room(1),room(5));
                goto 2 (door(2),room(5));
                gothru(door(2),room(5),room(2))
```


## .1.4 Cominents

The times quoted are for orderings of axioms, goals and action preconditions exactly as above. Apart from the goals, and preconditions of 'turnon', these orderings have been carefully chosen to produce best results with the present depth-first search strategy.

It is interesting to note that, for this problem domain, there is a single "natural" order for action preconditions, corresponding to the order in which one would expect them to be achieved. Thus the preconditions for 'turnon' are not stated in the natural order of:-

```
nextto(box(1),1ightswitch(S)) & on(robot,box(1))
```

Nevertheless WARPLAN still manages to find the optimal solution to 'status(1ightswitch(1),on)', unlike STRIPS.

In the solution for problem (4), WARPLAN generates the superfluous step marked with an asterisk, owing to the flaw mentioned in Section 5 concerning non-ground terms.
. 2 A Version of the Keys and Boxes Problem (of D. Michie).
.2.1 Description
This is a substantially simplified version of a "benchmark test" of Michie $\{11\}$.


The world comprises two areas 'inside' and 'outside'. There are four distinct locations 'inside', namely 'table', 'box 1', 'box 2', 'door'. There is a robot which is able to move about and transport objects. If the robot attempts to pickup an object at a location, all that can be ascertained is that. it will be holding one of the objects, if any, at that location. In our simplified formulation, the robot is only allowed to pickup an object if it is the only object at a location. There are two actions 'go' and 'take'; (for technical reasons, explained later, there are two versions of 'take'). The robot is only allowed to 'go' or 'take' something 'outside' if both the objects 'key 1' and 'key 2' are at the 'door'. In the
initial state 'key 1', 'key 2', 'red 1' are the only objects at, respectively, 'box 1 ', 'box 2 ', 'door' and nothing is at the 'table'. The goal is to have 'red 1' 'outside'.

## .2.2 Database

```
+ operator('is',righttoleft,4).
+ operator('set',prefix,5).
+ operator('placed',prefix,5).
+ operator('only',prefix,5).
+ operator('at',righttoleft,6).
+ add( position is P, go (P) ).
+ add( X is placed Q, take(X,P,Q) ).
+ add( set placed P is nothing, take(X,P,Q) ).
+ add( set placed Q is only X, take l(X,P,Q)).
+ add( Z,take 1 (X,P,Q)) - add(Z,take (X,P,Q)).
+ del( position is Z, go (P) ).
+ de1(X is placed Z, take ( X,P,Q) ).
+ del( position is Z, take ( X,P,Q) ).
+ del( set placed Q is Z., take (X,P,Q) ).
+ del( set placed P is Z, take (X,\Gamma,Q) ).
+ de1(Z , take 1 (X,P,Q) - deI (Z,take(X,P,Q) ).
+ can( go(inside at L-), true).
+ can( take(X,inside at L L ,inside at L L ),
        set placed inside at L}\mp@subsup{L}{1}{}\mathrm{ is only X&:
        position is inside at L L ).
+ can( take l (X,inside at L L ,inside at }\mp@subsup{L}{2}{}\mathrm{ ),
        set placed inside at L L is nothing &
        set placed inside at }\mp@subsup{L}{1}{}\mathrm{ is only X &:
        position is inside at L L ).
+ can( take(X,inside at L, outside),
        set placed inside at L is only X &
        position is inside at L 溇
        keyl is placed inside at door &
        key2 is placed inside at door).
```

+ always(true).
+ given (start kb, set placed inside at table is nothing ). + given(start kb, set placed inside at box 1 is only key 1 ).
+ given (start kb, set placed inside at box 2 is only key 2 ).
+ given(start kb, set placed inside at door is only red 1 ).
+ imposs(position is $P$ \& position is $Q$ ? $P \neq Q$ ).


## -2.3 The Problem

- plans( red 1 is placed outside, startkb).

Ans: startkb;
go(inside at door);
take l(red 1, inside at door, inside at table);
go(inside at box 1 );
take (key 1, inside at box 1 , inside at door);
go(inside at box 2);
take (key 2, inside at box 2, inside at door);
go(inside at table);
take(red 1 , inside at table, outside)
Time: 29 secs.

## .2.4 Commentary

With the present depth-first search strategy, getting a solution in an acceptable time was dependent on:-
(1) the right orderings of action preconditions etc.;
(2) omitting certain unnecessary axioms, e.g.
$+\operatorname{add}($ position is $Q, \operatorname{tak} \in(X, P, Q))$.
(3) a formulation using 'take' rather than 'pickup' and 'letgo'.

The reason for the two versions of 'take' is that an action must have a unique set of preconditions, and the effects of a 'take' depend on the set of objects at the destination. This is symptomatic of an oversimplification in WARPLAN which can and should be rectified.
. 3 Machine Code Generation

## -3.1 Description

I invented the following example to demonstrate that the system is general purpose. The example suggests an interesting area for future applications.

There is a very simple computer comprising an accumulator and an unspecificd number of general purpose registers. There are just four instructions 'load', 'store', 'add', 'subtract'. T'o axiomatise the domain for WARPLAN, it was necessary to follow each instruction in an assembly language program by a comment. The comment is introduced by '!' and states the value which will be in the accumulator after the instruction has been executed. Such comments are often needed by human programmers too:

## .3.3 Database

```
+ operator ('!',righttoleft,4).
+ operator ('is',righttoleft,4).
+ operator ('+',lefttoright,5).
+ operator ('-',lefttoright,5).
+ operator ('load',prefix,5).
+ operator ('add',prefix,5).
+ operator ('subtract',prefix 5).
+ operator ('store',prefix,5).
+ operator ('reg',prefix,5).
```

+ add (acc is $V_{1}+V_{2}$, add $R \quad V_{1}+V_{2}$ ).
+ add ( acc is $V_{1}-V_{2}$, subtract $R!V_{1}-V_{2}$ ).
+ add (acc is V,
+ add ( reg $R$ is $V$,
store $R \quad$ ! $V$.
$+\operatorname{del}(\operatorname{acc} i s Z, U)$ - add (acc is. $V, U$ ).
$+\operatorname{del}(\operatorname{reg} R$ is $Z, U)$ - add ( $\operatorname{reg} R$ is $V, U$ ).
$+\operatorname{can}$ (load R : $V$, reg $R$ is $V$ ).
$+\operatorname{can}$ ( store $R \quad$ : $V$, acc is $V$ ).
$+\operatorname{can}$ ( add $R \quad: V_{1}+V_{2}$, reg $R$ is $V_{2} \rho_{i}$ acc is $V_{1}$ ).
$+\operatorname{can}$ ( subtract $R: V_{1}-V_{2}$, reg $R$ is $V_{2}$, acc is $V_{1}$ ).
+ given (init, reg 1 is cl).
+ given (init, reg 2 is c2).
+ given (init, reg 3 is c3).
+ given (init, reg 4 is c4).
.3.3 Problems
(1) - plans( acc is (c1-c2)+(c3-c4), init).

Ans: init;
load 3 !c3;
subtract 4 !c3-c4;
store $X_{1} \quad$ !c3-c4;
load 1 !c1;
subtract 2 (c1-c2;
add $X_{1} \quad!(c 1-c 2)+(c 3-c 4)$
(2) - plans( acc is ( $\mathrm{c} 1-\mathrm{c} 2$ ) $+(\mathrm{c} 1-\mathrm{c} 2)$, init).

Ans: init;
load 1 !cl;
subtract 2 !c1-c2;
store $X_{1} \quad$ !c1-c2;
add $X_{1} \quad!(c 1-c 2)+(c 1-c 2)$
(3) - plans( reg 1 is $c 1+(c 2-c 3) \&$
reg 2 is c2-c3\&
reg 3 is c4+c4, init).
Ans: init;
load 2 !c2;
subtract 3 !c2-c3;
store 2 :c2-c3;
load 1 !cl;
add 2 !c1+(c2-c3);
store $1 \quad$ !c1+(c2-c3);
load 4 !c4;
add $4 \quad$ :c4+c4;
store 3 !c4+c4
Time: 18 sec
(4) - plans ( reg 1 is $c 1+(c 2-c 3) \&$
reg 2 is c2-こ3 $\&$
acc is c1, init.).
Ans: init;
load 2 !c2;
subtract 3 !c2-c3;
store 2 :c2-c3;
load $1 \quad$ !c1;
store $X_{1} \quad!c 1 ;$
add $2 \quad$ :c1+(c2-c3);
store $1 \quad!c 1+(c 2-c 3)$;
load $X_{1} \quad$ :cl
Note. The first branch in WARPLAN's search space for this problem is infinite. Interactive intervention to block this branch resulted in the above solution being found without any further assistance.

## .3.4 Comments

Here WARPLAN is behaving as a simple "compiier-compiler" from PC to machine code using a machine definition also written in PC. It is interesting to note that a very uniform proof procedure with no special domain-dependent heuristics can generate nicely "optimised" code automatically, as first solution.

Of course, once again I have "cheated" slightly by taking advantage of the freedom to order clauses and the terms in a conjunction to get the best results.

## . 1 Introduction

PROLOG is an elegant and powerful programing language developed at the University of Marseille. It bears certain similarities with PLASNET. \{6\}. Alternatively, one may regard the PROLOG interpreter as an efficiert PC theorem prover for problems renameable as Horn ciauses. The present system \{1\} is implemented partly in FORTRAN, partly in PROLOG itself, and running on an IBM 360-67 achieves roughly 200 unifications per second. (An earlier version of PROLOG is described in \{2\}.)

The features of PROLOG used in WARPLAN are summarised below. I have modified the syntax in certain inessential respects to improve legibility and to assist Anglo-saxon readers. The chief differences in the implemented form are:-
(1) only upper-case is used, so variables are prefixed by an '*' (asterisk);
(2) certain names (of evaluable predicates) are, of course, their French equivalents (and may soon be changed anyway).

## . 2 Syntax

The syntax is essentially the same as PC in clausal form.
Positive literals are preceded by '+', negative by '-'. The literals of a clause are simply concatenated with no explicit sign for disjunction. Each clause is terminated by a '.'.

An identifier is either a sequence of alpha-numeric characters or a single non-alpha-numeric character. Identifiers for variables comnence with an upper case letter; other identifiers denote predicates, functions or constants according to context.

Certain functions may be specified as infix or prefix operators to improve readability. Terms containing such functions are converted to the standard form on input and (nice point) are converted back on output. An operator must be declared before it is first used, by an axiom of the form:-

```
+ operator (F,D,P),
```

F is the name of the operator (function);
D is either 'prefix' or else specifies, for an infix operator, the direction of association (default bracketing).
$P$ is a number indicating the levnl of the operator in a precedence hierarchy.

Thus using the operators specified in Appendix I and Appendix III.3.2, the following terms are equivalent:-
reg 1 is c1+c2-c3 \& reg 2 is c4 \& acc is c1
reg 1 is ( $c 1+c 2$ ) -c 3 \& (reg 2 is $c 4{ }_{c}^{\circ}$ acc is $\dot{c} 1$ )


## . 3 Semantics

The "denotational" semantics is essentially the same as PC in clausal form.

The "operational" semantics, or proof procedure of the PROLOG interpreter, is as follows. The interpreter attempts to cancel (by resolution) the literals of a clause left-to-right, depth-first with backtracking. When attempting to cancel a literal, the candidate complementary literals are restricted to the leftmost literals of input clauses. The candidate input clauses are tried in the order in which they appear in the list of input clauses. Thus it is a linear inference system which may be described as SL resolution \{7\}, without merging, or ancestor resolution, with leftmost literal in a cell as selected, and with a depth-first search strategy. There is no "occur check" in unification. (Ancestor resolution and merging can be achieved by explicit programmer action.)

Certain predicates are defined as "evaluable", and behave as built-in procedures (frequently with side effects):-
'/' (slash): when this predicate is cancelled, it has the effect of prohibiting backtracking on any choices since (and including) the time the literal complementary to the leftmost literal in the input clause containing the '/' was cancelled. Thus it makes the choice of the current clause deterministic.
'plus $(X, Y, Z)$ ': adds the integers $X$ and $Y$ to yieid result $Z$.
'output $(X)^{\prime}:$ prints the term $X$ on the user's terminal,
'newline': causes a new line to be started on the user's terminal.
'univ $(X, Y)$ ' : takes a term $X$ and returns a list $Y$ of which the first element is the name of the principal function of $X$ (represented as a list of characters), and the remaining elements are that function's arguments in $X$. A list is constructed from the function '.' and constant 'nil' just as LISP uses CONS and NIL. e.g:-
univ (fun (a, $X$, foo ( $Y$ ) ) ,
. ( . (f, . (u, . (n, ni.1))),

- ( a ,
- ( x ,
- ( foo (y), nily))))
or given that '.' is declared as a right-to-left infix operator:-univ (fun ( $a, X$, foo ( $Y$ ) ),
(f.u.n.nil).a. X . foo (Y) .nil)

A variable may appear in place of a literal in a clause. This is analagous to allowing procedures to be a data-type in other programning languages. For an example of its use, see 'unless' in Appendix I.

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I have obviously drawn heavily on the ideas of Cordell Green and the designers of STRIPS.

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[^0]:    * For a definition of minimal (not to be confused with optimai) see Section 7.1.

[^1]:    * This requitenent is the fiot reasen why cercein artions need parameters which appear "unnecessary" irom the STRIPS stanapoint. (See Jection 3)

